CONCEPTUALIZING MATHEMATICALLY SIGNIFICANT PEDAGOGICAL OPENINGS TO BUILD ON STUDENT THINKING

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*The mathematics education community values using student thinking to develop mathematical concepts, but the nuances of this practice are not clearly understood. We conceptualize an important group of instances in classroom lessons that occur at the intersection of student thinking, significant mathematics, and pedagogical openings—what we call **M**athematically Significant Pedagogical **O**penings to Build on **S**tudent **T**hinking (MOSTs)—and introduce a framework for determining when they occur. We discuss how the MOST construct contributes to facilitating and researching teachers' mathematically-productive use of student thinking through providing a lens and generating a common language for recognizing and agreeing upon high-leverage student mathematical thinking.*

Research in mathematics teacher education suggests the benefits of instructional practices that build on student thinking (e.g., Fennema, et al., 1996; Stein & Lane, 1996), but such practices are complex and difficult both to understand and to enact (Ball & Cohen, 1999; Sherin, 2002; Silver, Ghousseini, Gosen, Charalambous, & Font Strawhun, 2005). Often opportunities to use student thinking to further mathematical understanding either go unnoticed or are not acted upon by teachers, particularly novices (Peterson & Leatham, 2009; Stockero & Van Zoest, 2012). Despite a growing number of teachers who are convinced of the value of student thinking and the need to encourage it, neither teachers nor those who educate them have a clear understanding of what thinking can best be used to develop mathematical concepts (Peterson & Leatham, 2009; Van Zoest, Stockero, & Kratky, 2010). We address this issue by providing a conceptual framework for thinking about the instances of student mathematical thinking that emerge while teaching. We refer to high-leverage instances of student thinking—those that have the most potential to increase student understanding of important mathematical ideas—as **M**athematically Significant Pedagogical **O**penings to build on **S**tudent **T**hinking (MOSTs).

We focus attention on the MOST construct because of its potential to contribute to the work of facilitating and researching teachers' mathematically-productive use of student thinking. Although this paper is about characterizing and recognizing MOSTs (as opposed to using them), they are better understood if the reader has a sense of our vision of how they might be productively used to support student learning. A teacher may respond to MOSTs in a variety of ways, from inserting a teacher explanation to asking follow-up questions to orchestrating a class discussion. When a teacher sees a MOST as an opportunity to step in and explain, it could be classified as naïve use (Peterson & Leatham, 2009) in that the teacher may be using the MOST merely as a trigger to lecture about the mathematical topic, rather than to build on the student

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thinking. A more productive use of a MOST is to orchestrate a discussion around the mathematics at hand. This orchestration could be done, for example, by posing questions that focus the class on connections between the mathematics of the observed student thinking and other concepts that are related to the mathematical goals of the classroom.

The MOST construct contributes to research on productive use of student mathematical thinking primarily through providing a lens and generating a common language for recognizing and agreeing upon high-leverage instances of student mathematical thinking. Specifically, it contributes to the work of facilitating teacher learning by providing guidance for identifying the characteristics of students' mathematical thinking that are most productive to focus on in preservice teacher coursework and inservice teacher professional development. It also provides a framework and language for conversation among teacher educators and teachers about high-leverage student thinking. Similarly, it contributes to researching teachers' use of student thinking by providing a lens to focus classroom discourse analysis on student mathematical thinking and tools to assess which student mathematical thinking is high-leverage. In this paper we describe the characteristics of MOSTs and introduce a framework for identifying them.

MATHEMATICALLY SIGNIFICANT PEDAGOGICAL OPENINGS TO BUILD ON STUDENT THINKING

Although skilled teachers and teacher educators often recognize when important mathematical moments occur during a lesson and can readily produce ideas about how to capitalize on them, the literature reveals a construct that is neither well-defined nor explicitly articulated. While not the focus of extant literature, such instances are mentioned in a number of different ways. For example, Jaworski (1994) referred to "critical moments in the classroom when students created a moment of choice or opportunity" (p. 527). Davies and Walker (2005) used the term "significant mathematical instances" (p. 275) and Davis (1997) used "potentially powerful learning opportunities" (p. 360). Schoenfeld (2008) referred to moments that contained "the fodder for a content-related conversation" (p. 57), "an issue that the teacher judges to be a candidate for classroom discussion" (p. 65) and the "grist for later discussion or reflection" (p. 70). Schifter (1996) spoke of "novel student idea[s] that prompt teachers to reflect on and rethink their instruction" (p. 130).

It is clear from the literature that these instances, whatever they are called, are important to mathematics teaching and learning. In studying such references to these instances and drawing on our own classroom and research experiences, we have identified three critical characteristics of these moments: student thinking, significant mathematics, and pedagogical openings.

Student Thinking

Because the MOST construct is designed to help articulate productive use of student mathematical thinking, we begin by defining what we mean by *student thinking*. We recognize our inability to access directly the thoughts of students. Instead we make inferences based on our observations of what they say and do. Teachers (and researchers) must "listen to the student, interpret what the student does and says, and try to build a 'model' of the student's conceptual structures" (von Glasersfeld, 1995, p. 14). Thus, when we use the phrase *student thinking* we refer to observable evidence of student thinking, which we define as any instance where a student's words or actions provide sufficient evidence to make reasonable inferences about their thinking. In the classroom setting, this evidence most commonly is visible in verbal utterances, gestures, or written work (including on the board).

Note that we make a distinction between *observable* and *observed*. There are many cases, particularly with novice teachers, where student thinking is observable, but not observed by the teacher (e.g., Peterson & Leatham, 2009; Stockero & Van Zoest, 2012). One explanation for this phenomenon is *inattentional blindness* (Simons, 2000)—described in the psychology literature as a failure to focus attention on unexpected events. In addition, these ideas are closely tied to teacher noticing (e.g., Sherin, et al., 2011)—what a teacher attends to (or fails to attend to) during a lesson. In the context of teaching, the teacher's failure to observe student thinking may mean that the teacher is not paying attention to student thinking or does not notice a particular instance of student thinking, rather than that there is no observable evidence of student thinking. Thus, for the purposes of our work, *observable* refers to thinking that could be observed by someone (e.g., the teacher, other students, a researcher) who witnessed the instance, either by being present or by engaging with a record of the interactions.

Mathematically Significant

In order to be a MOST, the mathematics in an instance must warrant use of limited instructional time; that is, it must be what we call *mathematically significant*. We use the term *mathematically significant* in the context of teachers engaging a particular group of students in the learning of mathematics. Thus, we see it as a subset of important mathematics, which can be determined apart from a specific classroom context. In the mathematical analysis of an instance, we consider mathematically significant in relationship to three key criteria: the importance of the mathematical idea of the instance, the appropriateness of the mathematics to the students in the classroom, and the extent to which the mathematics is connected to the mathematical goals for this group of students.

To determine whether the **important mathematics** criterion is met one must first determine whether the student thinking is mathematical in nature and, if so, what mathematics the student is expressing—what we call *the mathematics of the instance*. In order to determine the importance of the mathematics of the instance, one must be able to articulate an important mathematical idea that is closely related to the mathematics of the instance. Because this determination is purely mathematical, it can be made independent of a particular classroom context.

A second criterion for mathematically significant is that the mathematics of the instance be **appropriate** for the students in the classroom. That is, it must help students develop mathematically and move forward in their learning. Meeting this criterion requires two things. First, the mathematics of the instance must be accessible to the students given their prior mathematical experiences; they must have adequate background knowledge to engage with the mathematical idea. Second, the students must not yet have mastered the mathematical idea related to the mathematics of the instance. If they had, pursuing that idea would not likely move them forward in their learning. Thus, the appropriate mathematics criterion requires that the mathematical idea be accessible to students with a particular level of mathematical experience while not being likely to have been already mastered.

A third criterion of mathematically significant is that there is a viable mathematical connection between the mathematical idea related to the instance and **mathematical goals** for student learning in that class. The mathematical goals for the classroom encompass both mathematical content and mathematical practices. They could be determined by the teacher or by an external source, such as curriculum documents, or they could be inferred by an observer who is knowledgeable in the field of mathematics education, such as another teacher, a researcher or a teacher educator. When analyzing the mathematical idea related to an instance in relation to the mathematical goals for student learning, it is important to consider a range of goals, from those for the lesson in which the instance occurs, to those for the unit of instruction in which the lesson occurs, for the course students are taking, or for their broader mathematical learning. In the case of lesson goals, the instance may focus on a particular mathematical idea or connections among ideas within a lesson. In the other cases, the instance might involve making connections to other areas of mathematics, revisiting ideas from prior courses, or previewing ideas from future courses. Developing mathematical ways of thinking could be goals at any of these levels.

Pedagogical Opening

Conscientious teachers continuously seek evidence of their students' engagement with a wide variety of instructional goals. They take cues from actions big and small, making adjustments and pushing students to elaborate, explain and justify their thinking. Not all student actions, however, are "critical moments" (Walshaw & Anthony, 2008, p. 527) that create "potentially powerful learning opportunities" (Davis, 1997, p. 360). In the interest of differentiating student actions that meet this higher threshold, we define *pedagogical openings* as observable student actions that provide compelling opportunities to work toward an instructional goal. To determine whether an opening has been presented one must consider both the *positioning* and the *timing* of an observable student action.

Building on the notion from the discourse analysis literature in general and the work of Harré (e.g., Davies & Harré, 1990) in particular, we define *positioning* as the way in which an observable student action positions that student with respect to the content of an instructional goal. Students are positioned well with respect to an instructional goal when they engage "deeply" with the content of that goal as opposed to "at a surface level." Whereas good positioning is determined by a *particular* student's engagement with the content of an instructional goal, good timing is determined with respect to the preparation of the class as a whole to engage with the idea being raised in ways that support, rather than supplant, overall instructional goals.

PUTTING THE THEORY INTO ACTION

When determining whether a MOST has occurred, the focus of our analysis is an "instance"—an observable student action or small collection of connected actions (such as a verbal expression combined with a gesture). Typically an instance is one conversational turn or physical expression (such as writing a solution on the board), but it can involve multiple turns. For example, if the expression of an idea were interrupted by another speaker with a comment that merely encouraged the initial speaker (e.g., "yeah," "okay," or "um-hum"), the speakers' initial idea and the continuation of it would be considered a single instance. Determining whether an instance qualifies as a MOST involves a systematic analysis of whether the instance embodies the three MOST characteristics (see Figure 1). This analysis begins with questioning whether the instance provides observable evidence of student thinking. If it does not, the analysis ends because the instance cannot be a MOST. Focusing first on this characteristic stems from the perspective that what students say or do during a lesson is critical and should inform the teacher's actions. If observable evidence of student thinking is present, the mathematics of the instance is then analyzed to determine whether the instance is mathematically significant; that is, whether it satisfies the important mathematics, appropriate mathematics and mathematical goals criteria. This mathematical analysis takes place linearly; if any mathematics criterion is not met, the analysis ends. The instance is not mathematically significant and therefore not a MOST. This mathematical analysis of the instance distinguishes our work from more general work on classroom discourse or even "teachable moments" in that we focus on instances that are likely to advance students' development of mathematical ideas. If the instance is determined to be mathematically significant, the instance is analyzed in terms of whether the positioning and timing are right to create a pedagogical opening. Again, if either criterion is not met, the analysis ends; if both are met, the instance has met the criteria for all three characteristics and is deemed to be a MOST. We have found that taking this flowchart approach to the analysis of an instance brings structure and simplicity to an often chaotic and complex task.

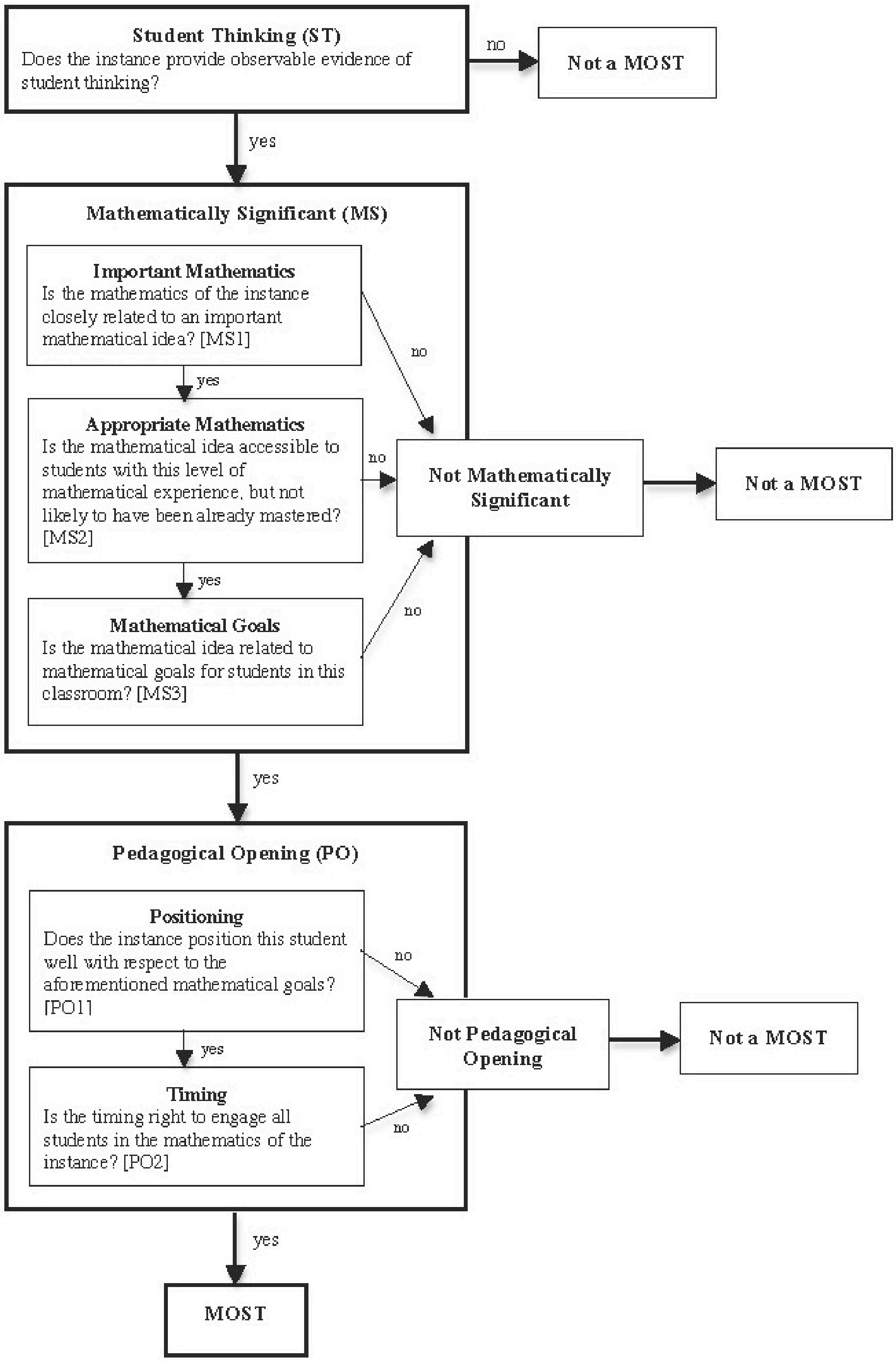

Figure 1. Analysis process for determining whether a classroom instance is a MOST.

CONCLUSION

By clearly defining three critical characteristics that distinguish instances that provide high-leverage opportunities to advance students' mathematical understanding from those that do not, the MOST construct has the potential to become a tool to make sense of classroom interactions. In particular, the construct provides both a means for systematically analysing instances of classroom discourse and a vocabulary for discussing the mathematical and pedagogical importance of student thinking that arises within such discourse. Considering whether an instance embodies the three characteristics of a MOST requires identifying the mathematics in an instance of observable student thinking, as well as the larger mathematical idea to which it is related. Instances that are determined to be mathematical are then framed in terms of both mathematical significance and the pedagogical opening they provide. Engaging in this analysis provides a mechanism for teacher educators and researchers to frame teachers' practice in terms of their use of high-leverage instances of student mathematical thinking. This framing shifts the focus of the work from *whether* a teacher is using student thinking, to *what* student thinking a teacher is incorporating into a lesson and *why* that incorporation is valuable.

Although we acknowledge that mathematics teachers', teacher educators' and researchers' considerations are influenced by a wide range of beliefs about the nature of mathematics and about its teaching and learning, as well as by their own mathematical knowledge, we present the MOST framework as a mechanism for building mutual recognition and appreciation of high-leverage opportunities to build on students' mathematical thinking. Engaging in discussions of instances of student thinking using a common language and framework provides an opportunity to advance understanding of the productive use of student mathematical thinking, and consequently, enhance the teaching and learning of mathematics.

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