

LeveragingMOSTs.org



Conceptualizing the Teaching Practice of Building on Student Mathematical Thinking

Laura R. Van Zoest

Western Michigan University

Keith R. Leatham & Blake E. Peterson

Brigham Young University

Shari L. Stockero

Michigan Technological University

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Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

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Using Student Mathematical Thinking in Instruction



- The mathematics education community has long encouraged instruction that uses students' mathematical thinking (e.g., NCTM, 1989, 2000, 2014)
- Not all student thinking has the same potential to support student learning (Leatham, Peterson, Stockero, & Van Zoest, 2015)
- The field does not have a common understanding of what it means to “use” student thinking (Leatham, Van Zoest, Stockero, & Peterson, 2014)
- The benefits of using students' mathematical thinking in instruction have been well-documented in certain areas (Fennema, et al., 1996; Franke & Kazemi, 2001; Smith & Stein, 2011; Stein & Lane, 1996)
- Incorporating in-the-moment student thinking into instruction has been understudied and has untapped potential

Principles Underlying Productive Use of Student Thinking



- Student mathematics is at the forefront
- Students are positioned as legitimate mathematical thinkers
- Students are engaged in sense making
- Students are working collaboratively

~aligned with NCTM, 2014, *Principles to Actions*

We see the *teaching practice of **building*** as simultaneously enacting these principles in response to student thinking.

Definition of Building



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Building

To make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

Basic building blocks



- an **instance of student thinking**: an observable student action or small collection of connected actions (such as a verbal expression combined with a gesture)
- **student mathematics (SM)**: the articulation of a reasoned inference about what the student is saying mathematically in the instance
- **mathematical point (MP)**: the articulation of the most closely related mathematical idea that can be gained from considering the instance of student thinking

The MOST framework (Leatham et al., 2015) identifies instances of student thinking worth building on

Building ...



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in Student
Thinking

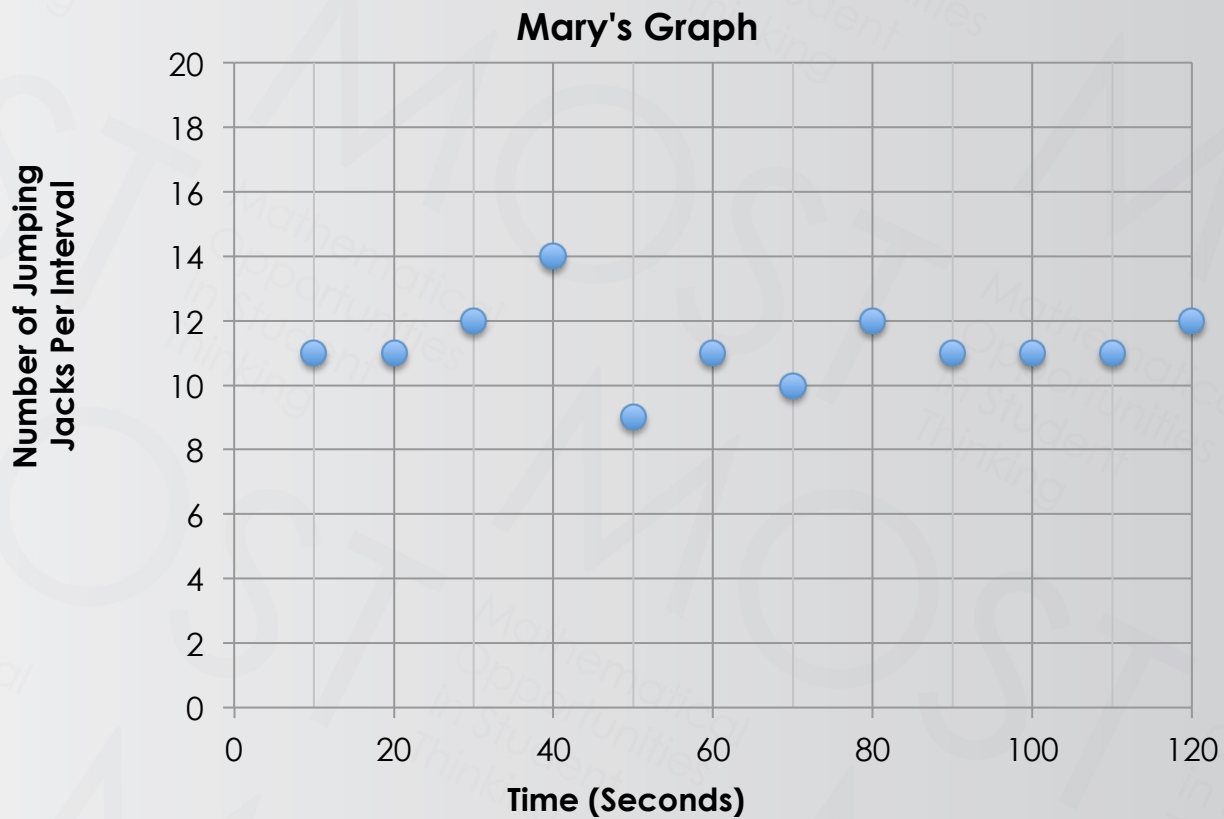
- is a complex practice
- is not a single move
- a collection of moves

What does building look like in action?

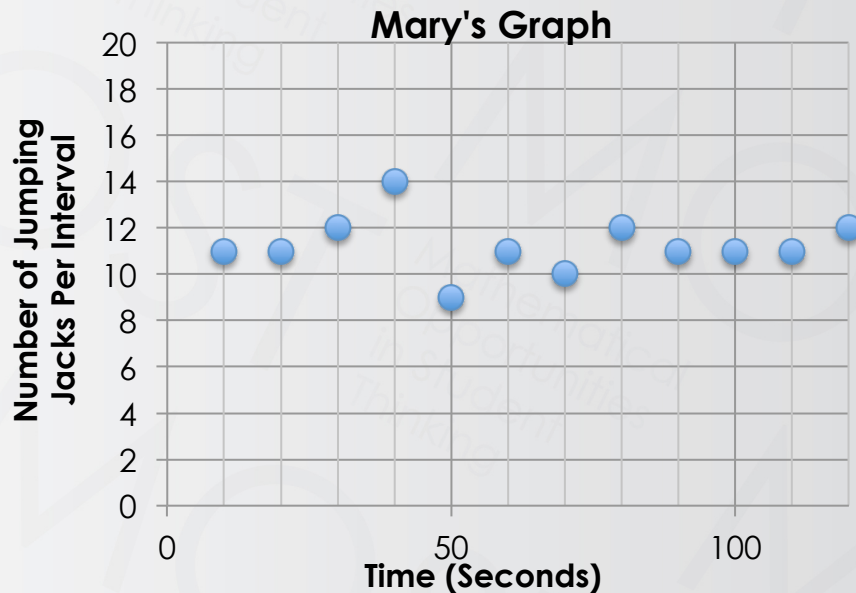
Jumping Jacks



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Jumping Jacks



Teacher: It says it on there—'per interval'—but what does that mean?

Student: Like um... I don't know but, right here, in 50 seconds she had only done 9 jumping jacks.

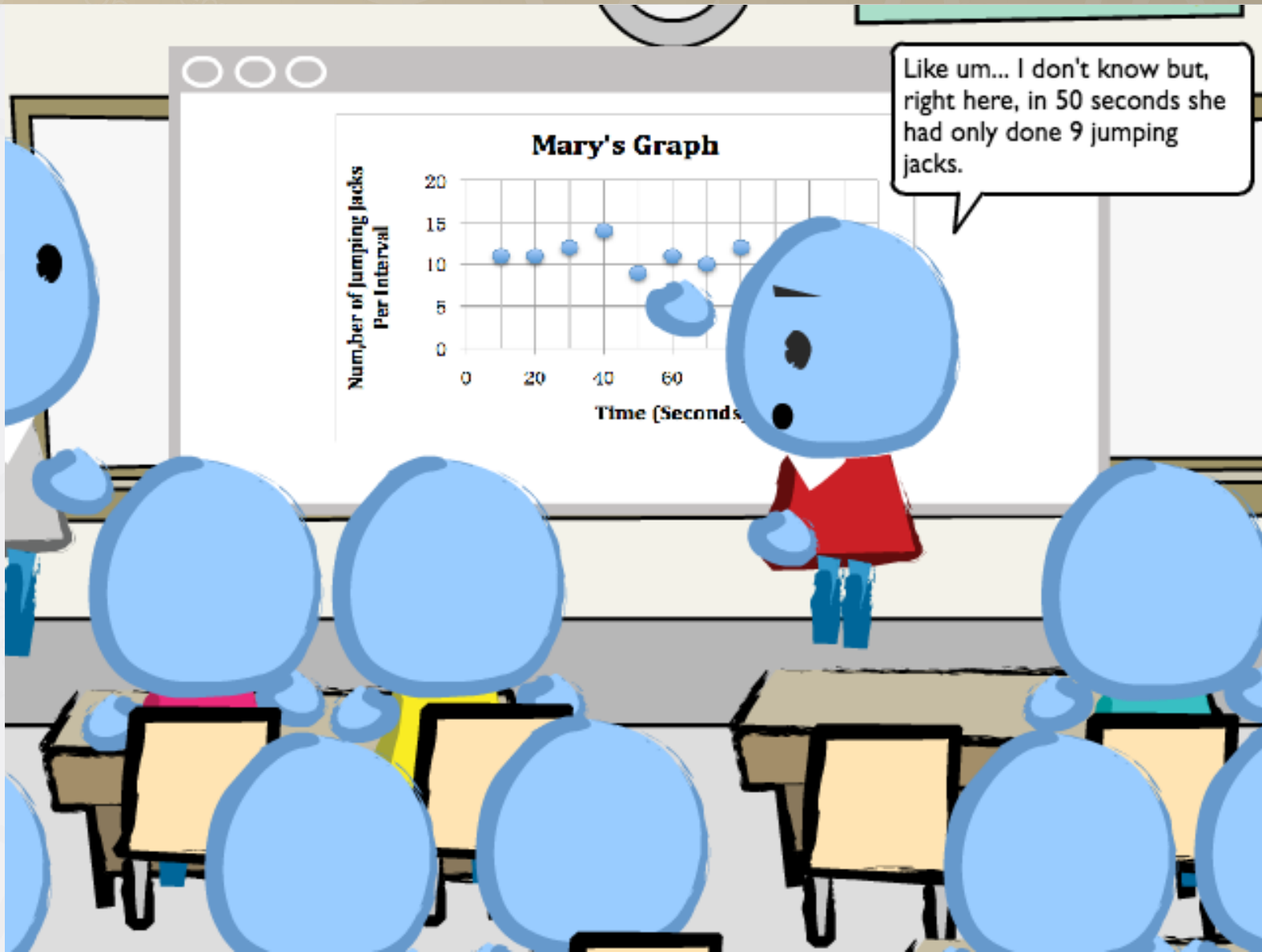
SM: The point (50, 9) on the graph means that in 50 seconds Mary had only done 9 jumping jacks.

MP: When measuring a quantity 'per interval' the dependent variable tells you how many units per interval (a rate) and not the total number of units.

Example



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Scenarios created with LessonSketch © The Regents of the University of Michigan (lessonsketch.org)



To make *student thinking* an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking

Building on Student Mathematical Thinking



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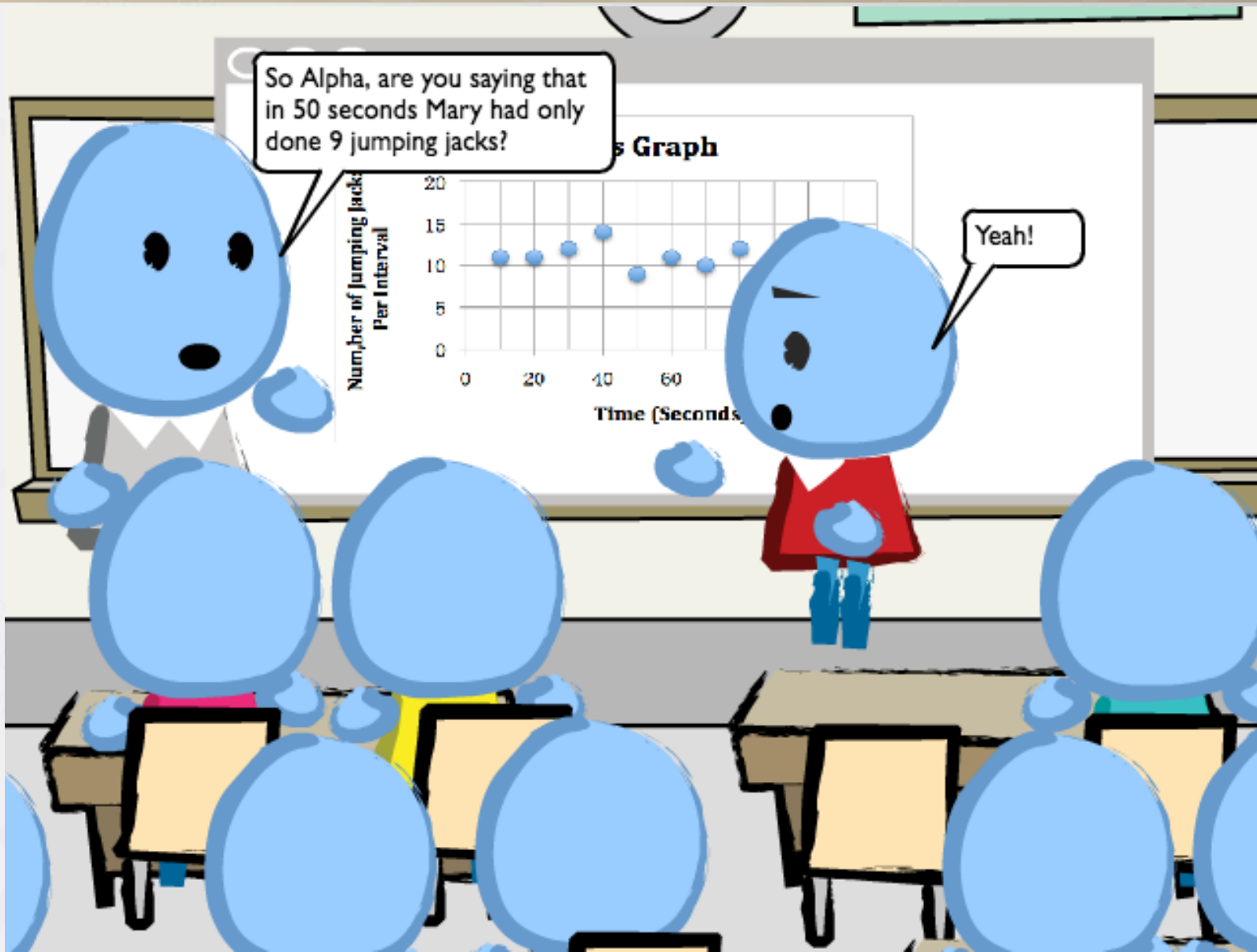
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0. Invite/allow students to share their mathematical thinking (*elicit*)

Example



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Building on Student Mathematical Thinking



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0. Invite/allow students to share their mathematical thinking (**elicit**)
1. Make the object of consideration clear

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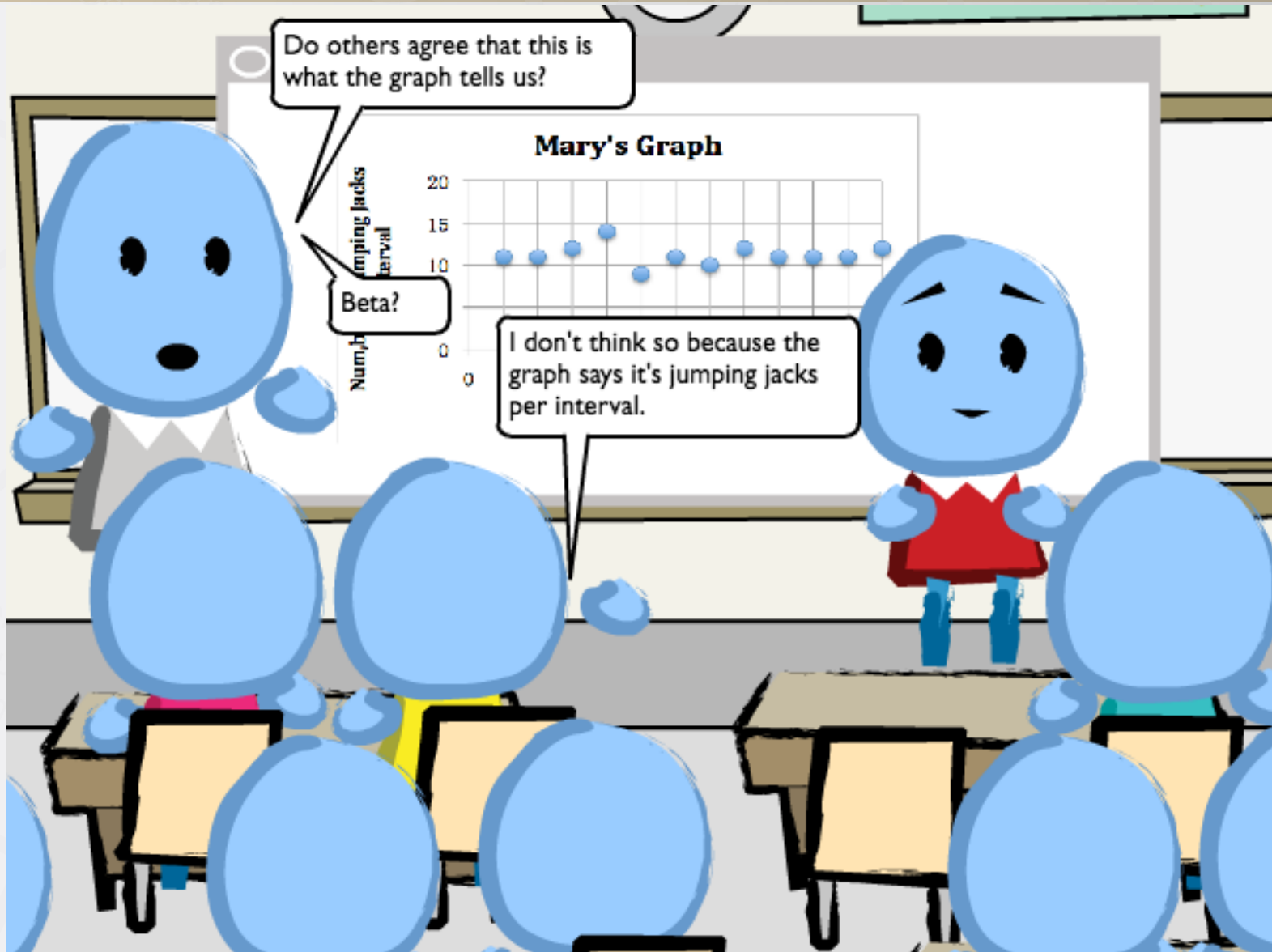


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2. Turn the object of consideration over to the students with parameters that put them in a sense-making situation

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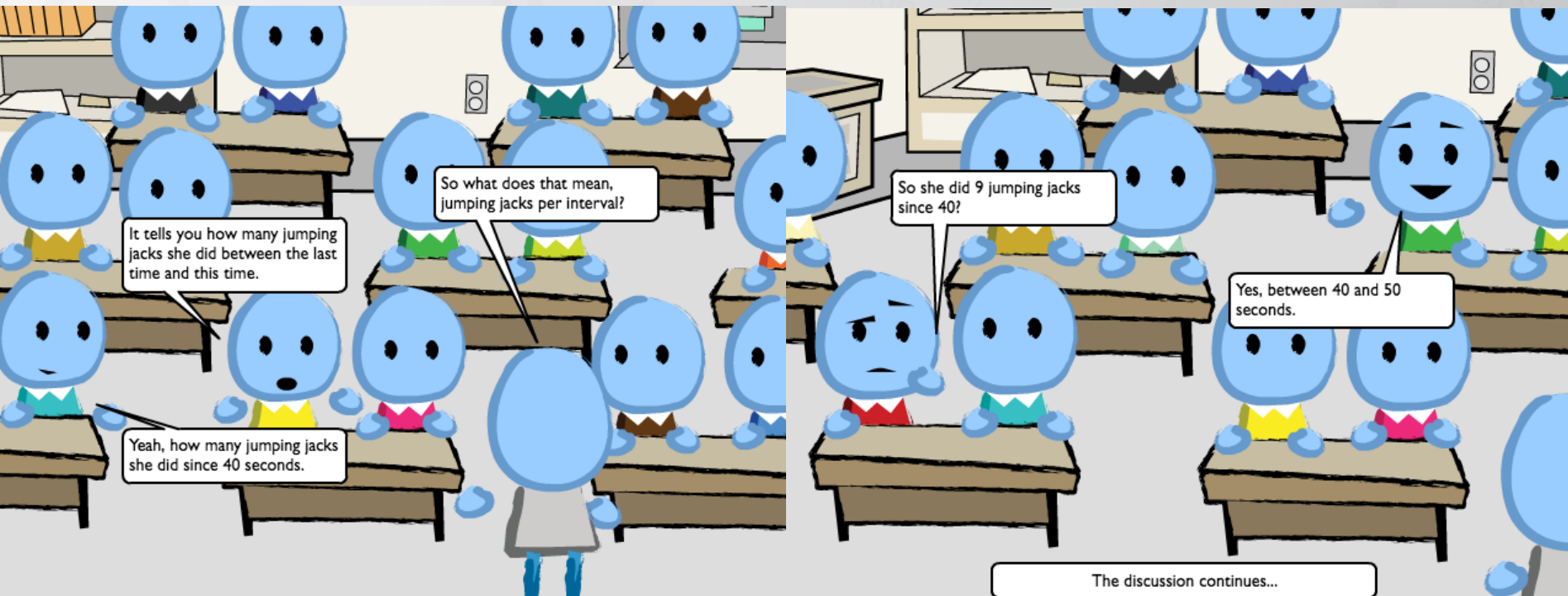


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Building on Student Mathematical Thinking



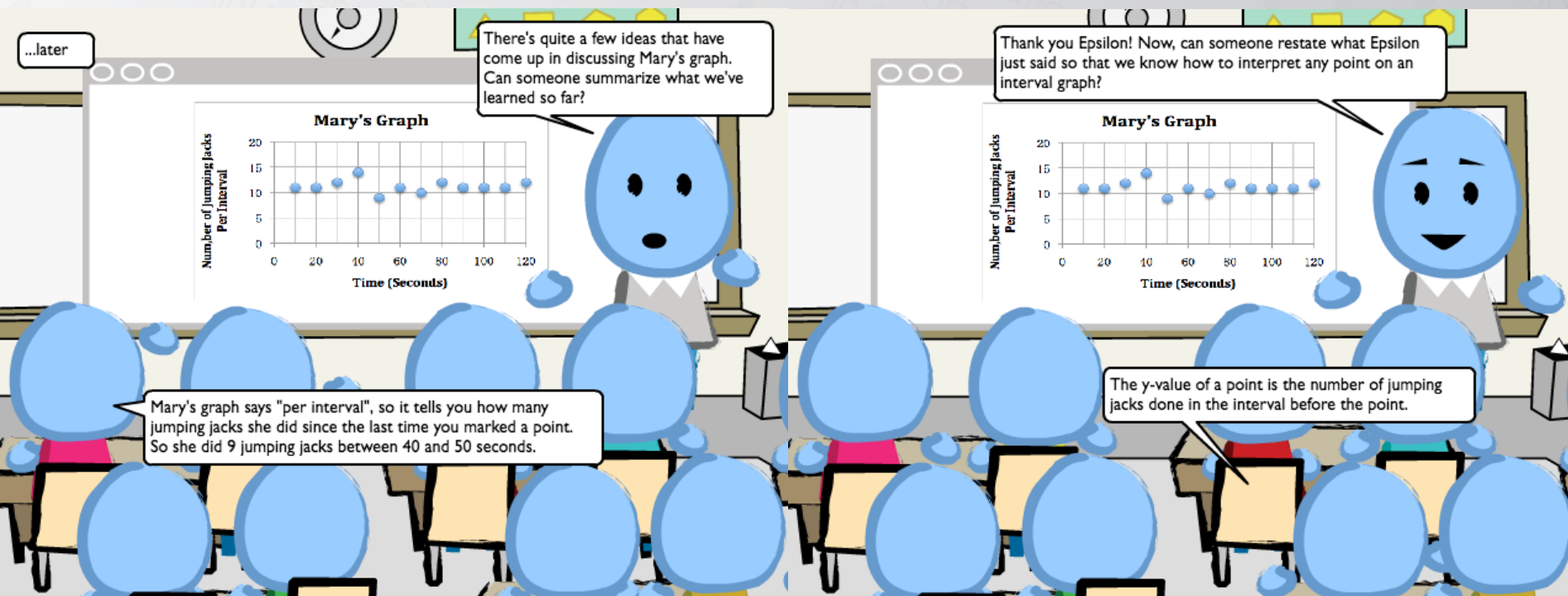
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Example

...later



There's quite a few ideas that have come up in discussing Mary's graph. Can someone summarize what we've learned so far?

Mary's Graph

Time (Seconds)	Number of Jumping Jacks Per Interval
0	10
10	11
20	12
30	13
40	14
50	9
60	10
70	11
80	12
90	11
100	12
110	11
120	12

Mary's graph says "per interval", so it tells you how many jumping jacks she did since the last time you marked a point. So she did 9 jumping jacks between 40 and 50 seconds.

Thank you Epsilon! Now, can someone restate what Epsilon just said so that we know how to interpret any point on an interval graph?

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The y-value of a point is the number of jumping jacks done in the interval before the point.

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4. Facilitate the extraction and articulation of the mathematical point of the object of consideration

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The Teaching Practice of Building

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Contact Information



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