Conceptualizing Mathematically Significant Pedagogical Opportunities to Build on Student Thinking

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Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

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Our Motivation for this Work

- Observed graduates of our teacher education programs who were successful in eliciting student thinking, but weren't using it to further students' mathematical understanding
- Saw "teachable moments" not get acted on
- Want to better understand these moments so we can prepare teachers to take advantage of them

From the literature…

- "critical moments in the classroom when students created a moment of choice or opportunity" (Jaworski, 1994, p. 527)
- "novel student idea[s] that prompt teachers to reflect on and rethink their instruction" (Schifter, 1996, p. 130)
- "potentially powerful learning opportunities" (Davis, 1997, p. 360)
- "significant mathematical instances" (Davies and Walker, 2005, p. 275)
- "[student's] comment provides the fodder for a content-related conversation" (Schoenfeld, 2008, p. 57)
- "crucial mathematic hinge moment[s]" (Thames and Ball, 2013, p. 31)

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3 Characteristics

- Students
- Mathematics
- Pedagogy

3 Characteristics

- Students—Student Thinking
- Mathematics
- Pedagogy

3 Characteristics

- Students—Student Thinking
- Mathematics—Mathematically Significant
- Pedagogy

3 Characteristics

- Students—Student Thinking
- Mathematics—Mathematically Significant
- Pedagogy—Pedagogical Opportunity

MOSTs

Mathematically significant
pedagogical **O**pportunity
to build on **S**tudent
Thinking

MOST Framework

- Our attempt to describe and identify MOSTs – to operationalize intuition and experience
- Two purposes
  - Our focus now is using it for research
  - Later we will modify it for use with teachers
- Focus of our analysis is an “instance” - an observable student action or small collection of connected actions (such as a verbal expression combined with a gesture)

3 Characteristics

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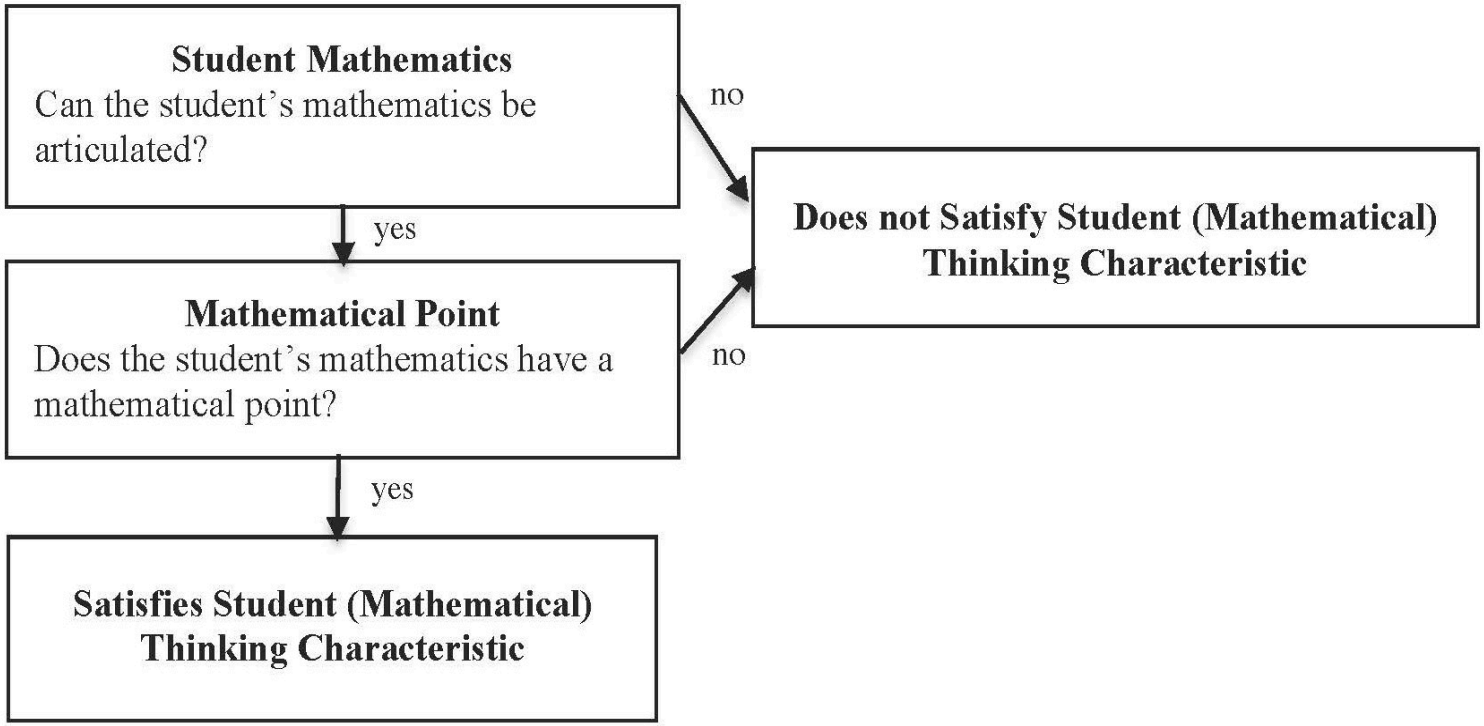
Student Mathematical Thinking

- An evidence-based inference about student mathematical thinking that can be used to develop a mathematical idea.
- In a classroom setting, evidence is most commonly visible in verbal utterances, gestures or written work (including on the board).
- Two criteria that must be met
  - Student Mathematics
  - Mathematical Point

Criteria for Student Mathematical Thinking

1. Student Mathematics
   - an inference (that can reasonably be made based on a student's actions) about what the student is thinking mathematically
2. Mathematical Point
   - a concise statement of a mathematical idea (related to the student mathematics) that a mathematics learner could know and understand


Student Mathematics
Can the student's mathematics be articulated?
no
yes
Does not Satisfy Student (Mathematical) Thinking Characteristic
Mathematical Point
Does the student's mathematics have a mathematical point?
no
yes
Satisfies Student (Mathematical) Thinking Characteristic

3 Characteristics

- Students—Student Thinking
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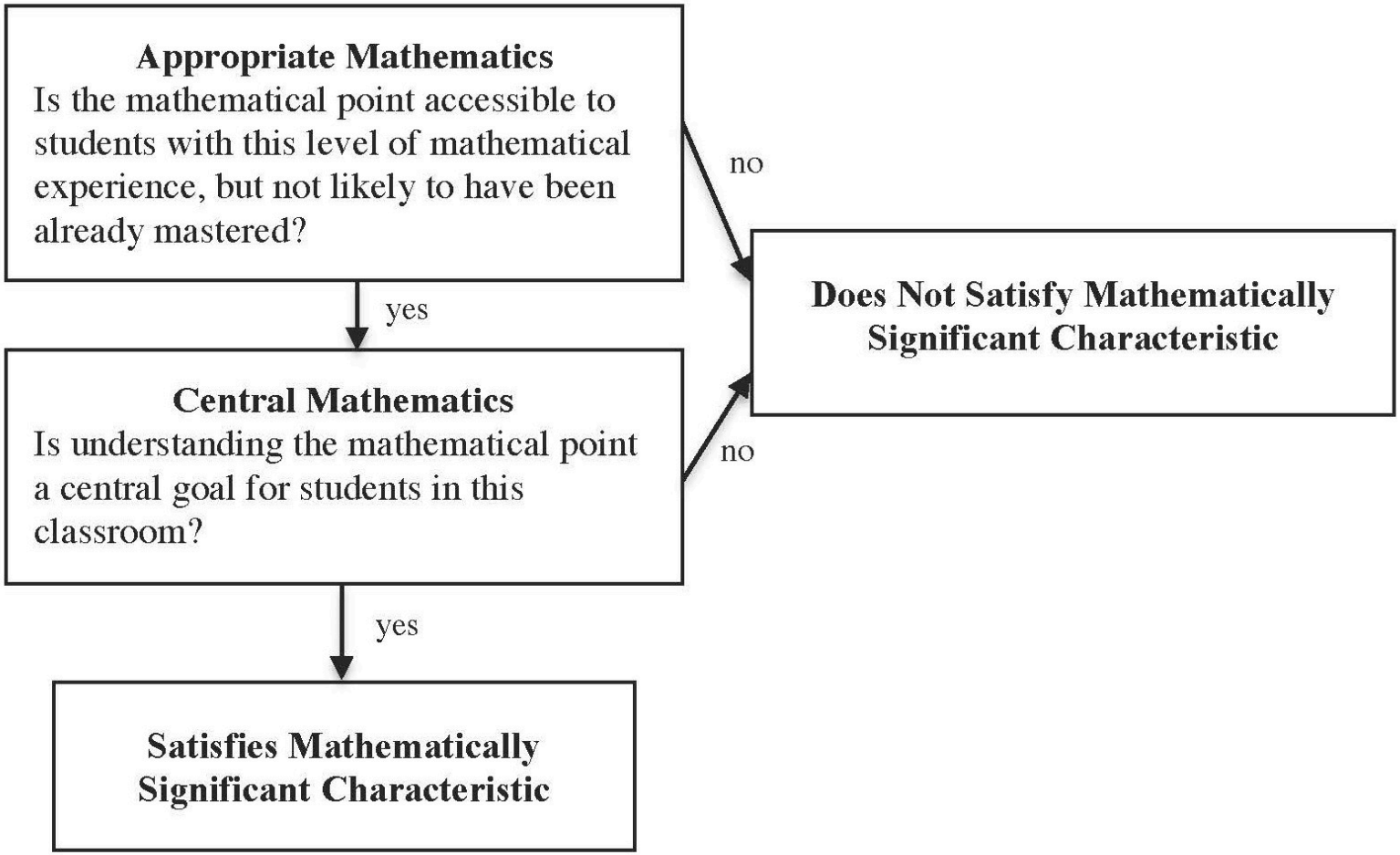
- Pedagogy—Pedagogical Opportunity

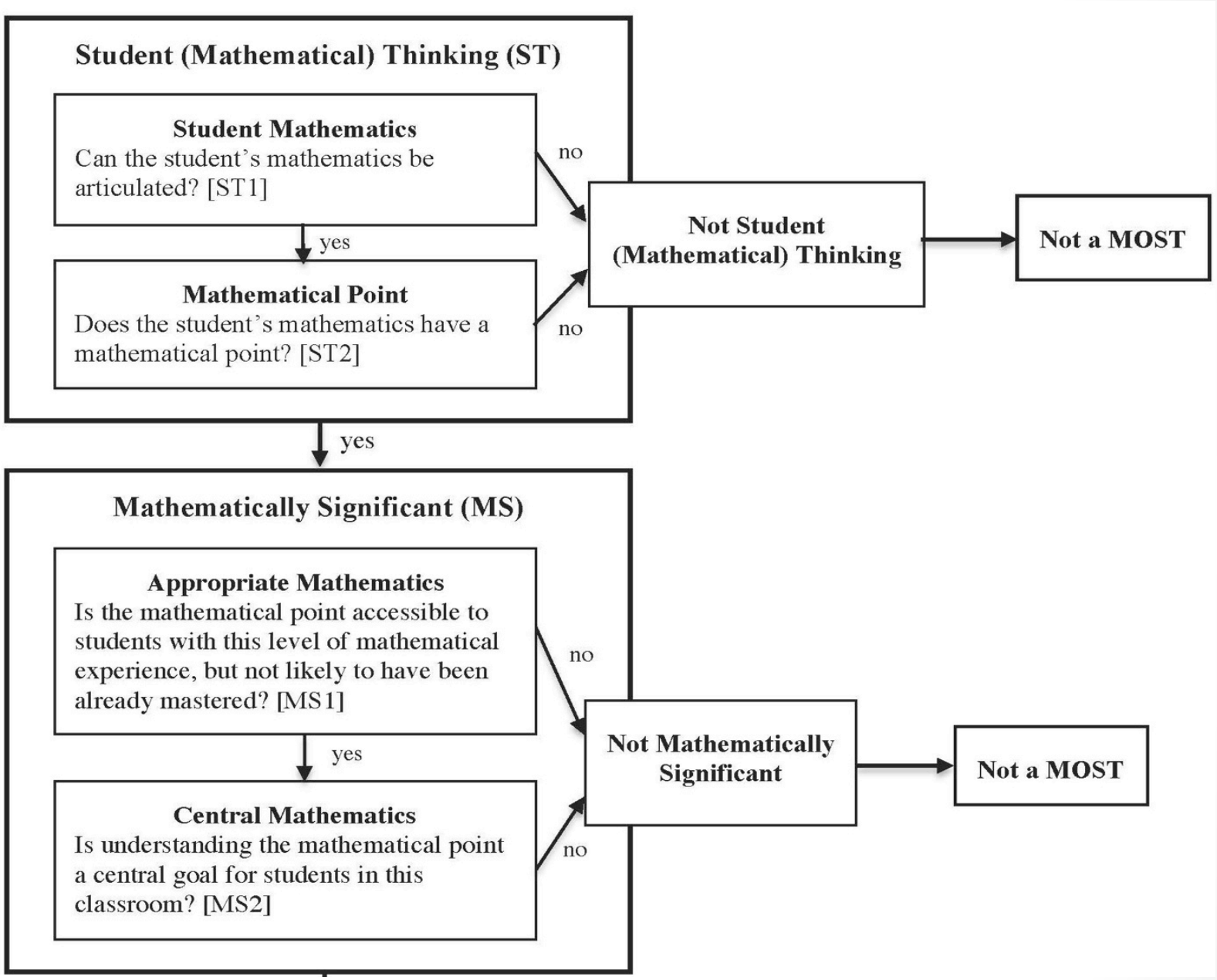
Mathematically Significant

- Must warrant use of limited instructional time
- Used in the context of teachers engaging a particular group of students in the learning of mathematics
- Two criteria that must be met
  - Appropriate Mathematics
  - Central Mathematics

Criteria for Mathematically Significant

1. Appropriate Mathematics
   - Accessible to students given their prior mathematical experiences
   - Not yet mastered
2. Central Mathematics
   - Understanding the mathematical point must be a central goal for this group of students
   - The goal could be central either to the lesson or to the discipline of mathematics


Appropriate Mathematics
Is the mathematical point accessible to students with this level of mathematical experience, but not likely to have been already mastered?
no
yes
Central Mathematics
Is understanding the mathematical point a central goal for students in this classroom?
no
yes
Does Not Satisfy Mathematically Significant Characteristic
Satisfies Mathematically Significant Characteristic


Student (Mathematical) Thinking (ST)
Student Mathematics
Can the student's mathematics be articulated? [ST1]
no
yes
Mathematical Point
Does the student's mathematics have a mathematical point? [ST2]
no
Not Student (Mathematical) Thinking
Not a MOST
yes
Mathematically Significant (MS)
Appropriate Mathematics
Is the mathematical point accessible to students with this level of mathematical experience, but not likely to have been already mastered? [MS1]
no
yes
Central Mathematics
Is understanding the mathematical point a central goal for students in this classroom? [MS2]
no
Not Mathematically Significant
Not a MOST

3 Characteristics

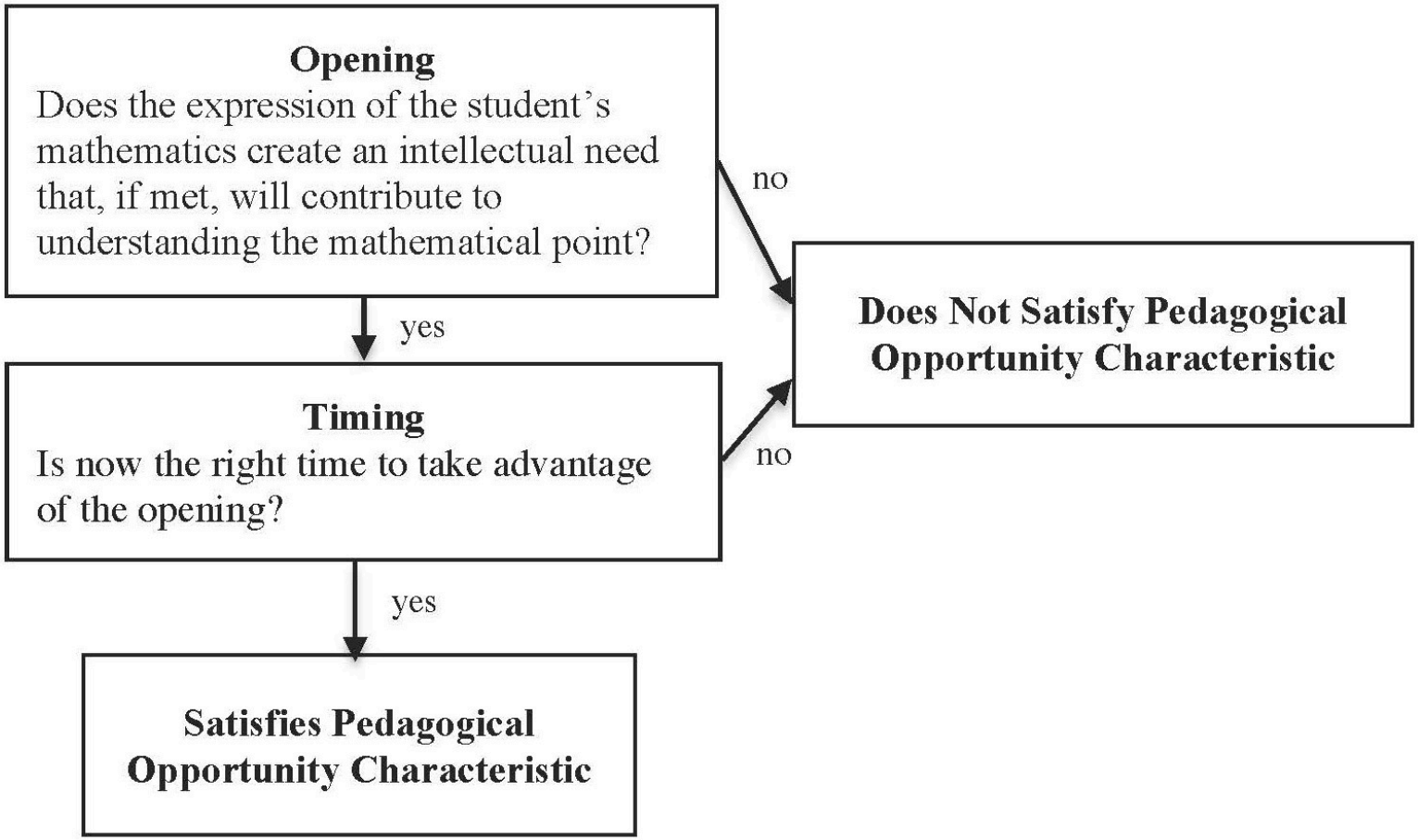
- Students—Student Thinking
- Mathematics—Mathematically Significant
- Pedagogy—Pedagogical Opportunity

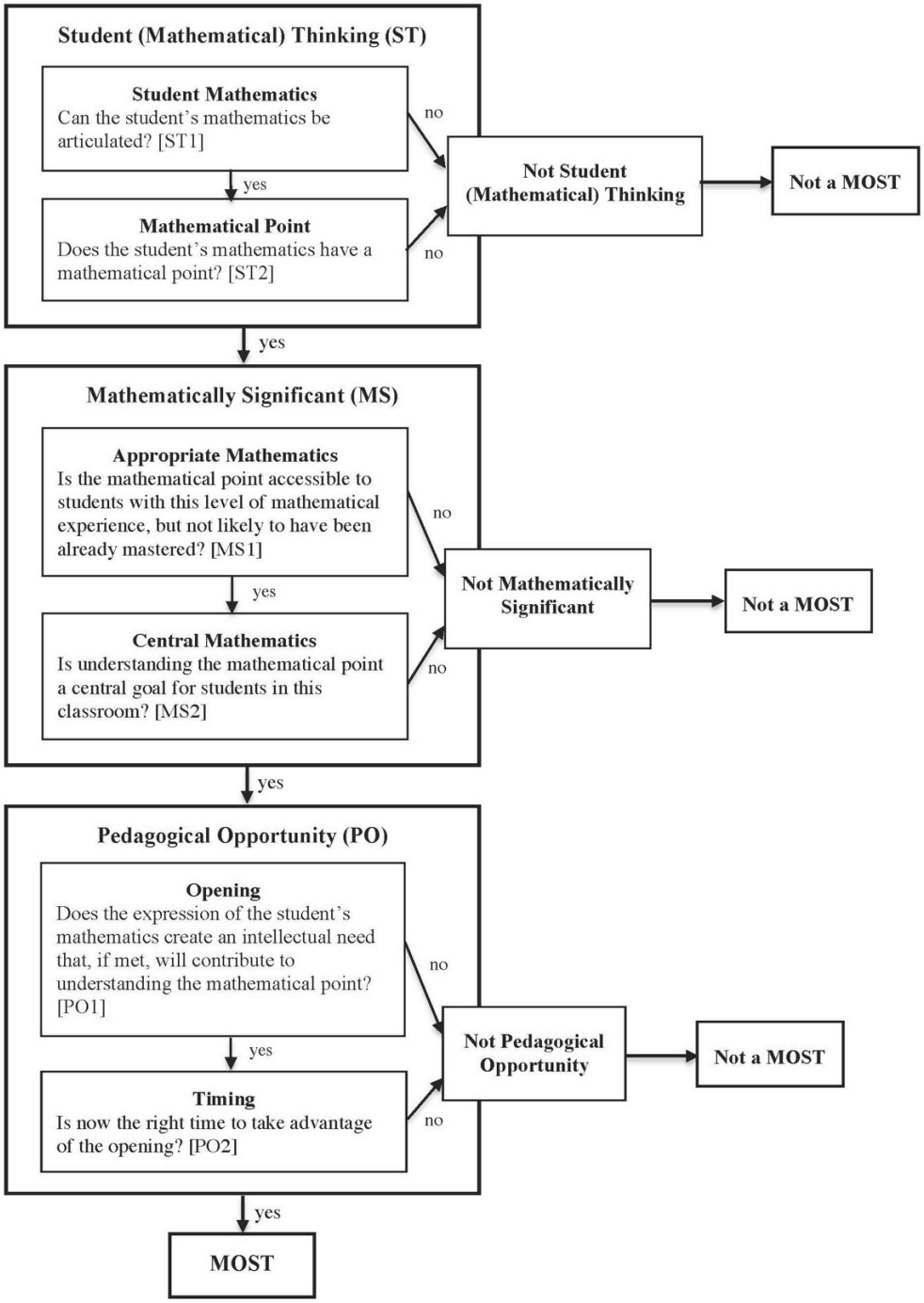
Pedagogical Opportunity
(to build on student thinking)

- An observable student action that creates an *intellectual need* (Harel, 2013) that can be acted on in that moment to contribute to students' understanding of a mathematical point.
- Two criteria that must be met
  - Opening
  - Timing

Criteria for Pedagogical Opportunity

1. Opening
   - an instance in which the expression of a student's mathematical thinking creates an intellectual need for students to make sense of that student's mathematics, providing an opportunity to understand the mathematical point of the instance
2. Timing
   - an opportune time to capitalize on the opening to help students understand the mathematical point of the instance


Opening
Does the expression of the student's mathematics create an intellectual need that, if met, will contribute to understanding the mathematical point?
no
yes
Timing
Is now the right time to take advantage of the opening?
no
yes
Does Not Satisfy Pedagogical Opportunity Characteristic
Satisfies Pedagogical Opportunity Characteristic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Appropriate Mathematics
Is the mathematical point accessible to students with this level of mathematical experience, but not likely to have been already mastered? [MS1]
no
yes
Central Mathematics
Is understanding the mathematical point a central goal for students in this classroom? [MS2]
no
Not Mathematically Significant
Not a MOST
yes
Pedagogical Opportunity (PO)
Opening
Does the expression of the student's mathematics create an intellectual need that, if met, will contribute to understanding the mathematical point? [PO1]
no
yes
Timing
Is now the right time to take advantage of the opening? [PO2]
no
Not Pedagogical Opportunity
Not a MOST
yes
MOST

Advantages of the MOST Flowchart

- Begins with the core of a MOST – student thinking
- Directs attention to our main focus –significant mathematics
- Forces the pedagogical opportunities identified to be those that use student thinking to further mathematical understanding

Questions?

Example Instance

During a grade 9 Algebra 1 lesson, the class is exploring how the “m” and “b” in the equation $y = mx + b$ are related to the graph of a linear function. During a class discussion about these relationships, a student asks if it is possible for the graph to have “two dots on the *y*-axis”—that is, whether is it possible for the linear function to have more than one *y*-intercept.

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