Mathematically Significant Pedagogical Opportunities to Build on Student Thinking

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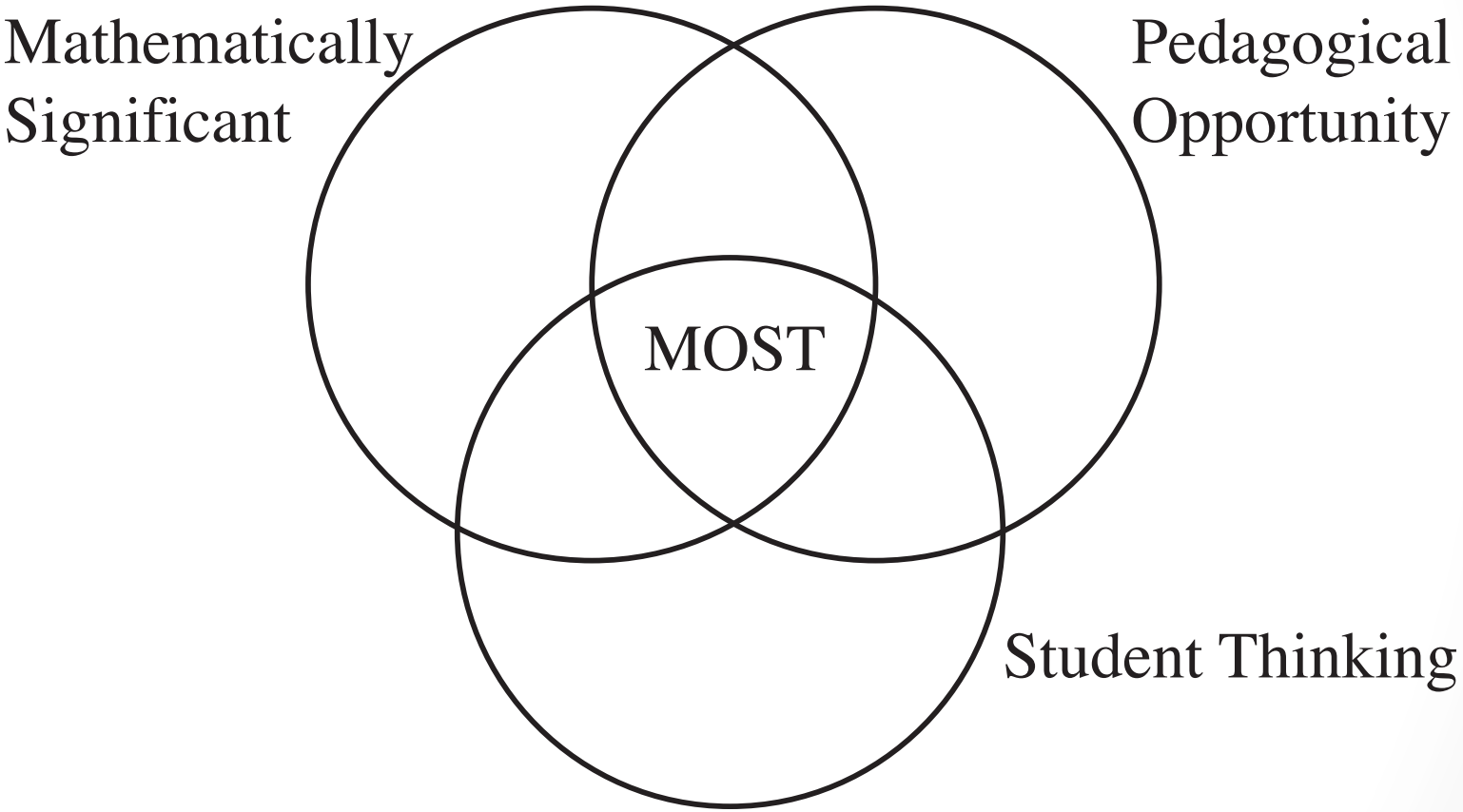
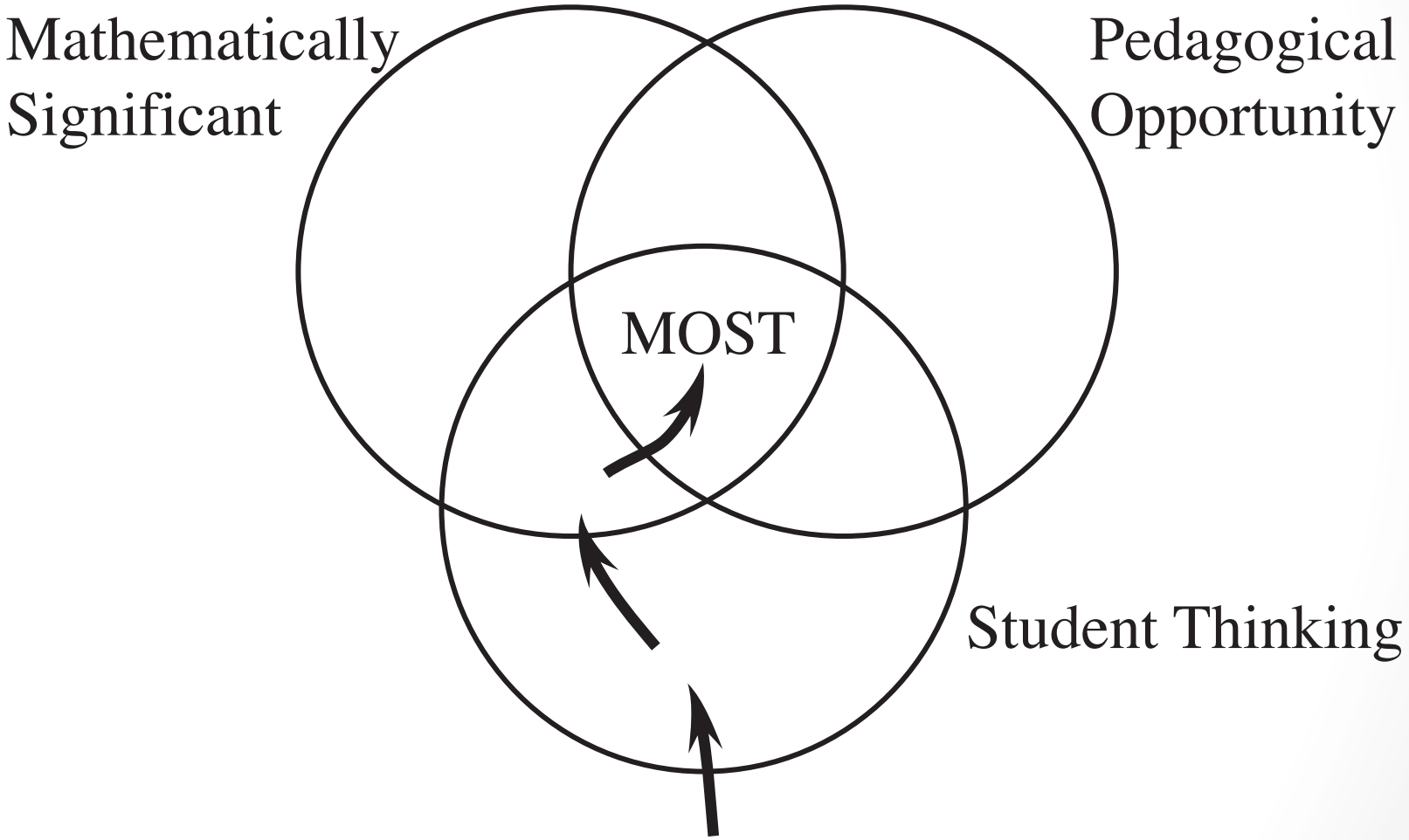
MOST

Mathematically significant
pedagogical **O**pportunity
to build on **S**tudent
Thinking

Relationship of MOST Characteristics



Pathway for Defining the MOST Characteristics



MOST Framework

- Our attempt to describe and identify MOSTs – to operationalize intuition and experience
- Two purposes
  - Our focus now is using it for research
  - Later we will modify it for use with teachers
- Focus of our analysis is an “instance” - an observable student action or small collection of connected actions (such as a verbal expression combined with a gesture)

3 Characteristics

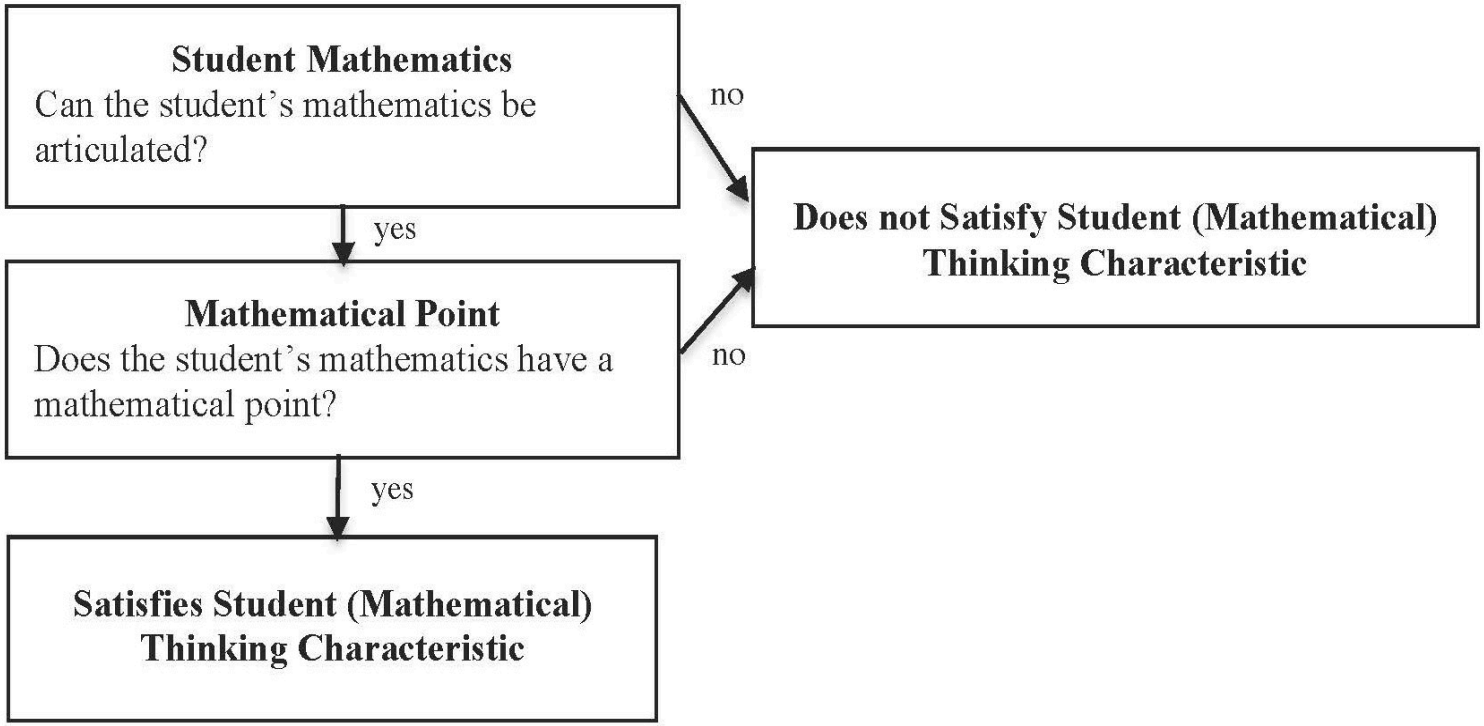
- **Student Thinking**
- Mathematically Significant
- Pedagogical Opportunity

Student Mathematical Thinking

- An evidence-based inference about student mathematical thinking that can be used to develop a mathematical idea.
- In a classroom setting, evidence is most commonly visible in verbal utterances, gestures or written work (including on the board).
- Two criteria that must be met
  - Student Mathematics
  - Mathematical Point

Criteria for Student Mathematical Thinking

1. Student Mathematics
   - an inference (that can reasonably be made based on a student's actions) about what the student is thinking mathematically
2. Mathematical Point
   - a concise statement of a mathematical idea that mathematics learners could better understand as a result of making the student mathematics of the instance an object for consideration.


Student Mathematics
Can the student's mathematics be articulated?
no
yes
Does not Satisfy Student (Mathematical) Thinking Characteristic
Mathematical Point
Does the student's mathematics have a mathematical point?
no
yes
Satisfies Student (Mathematical) Thinking Characteristic

3 Characteristics

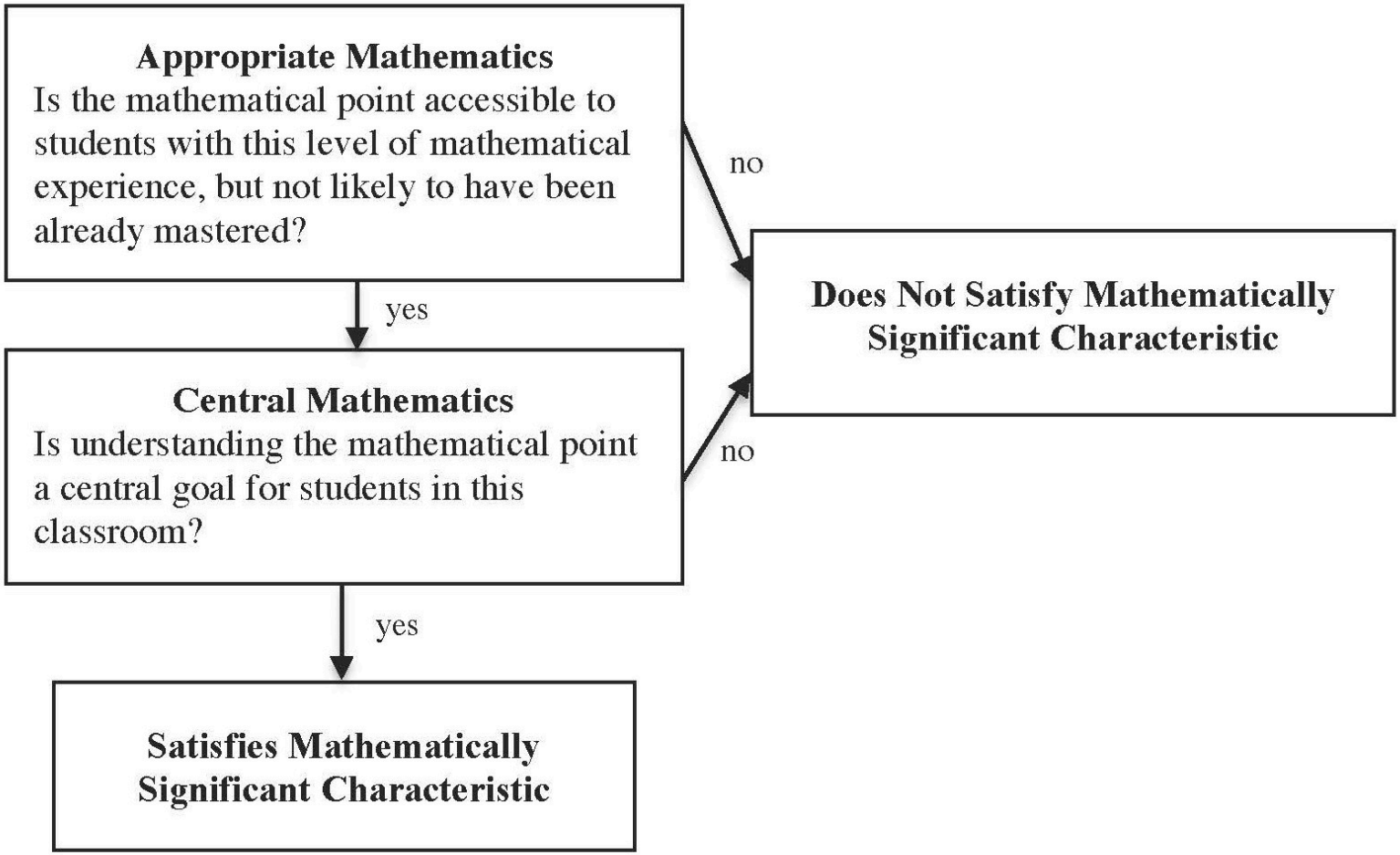
- Student Thinking
- **Mathematically Significant**
- Pedagogical Opportunity

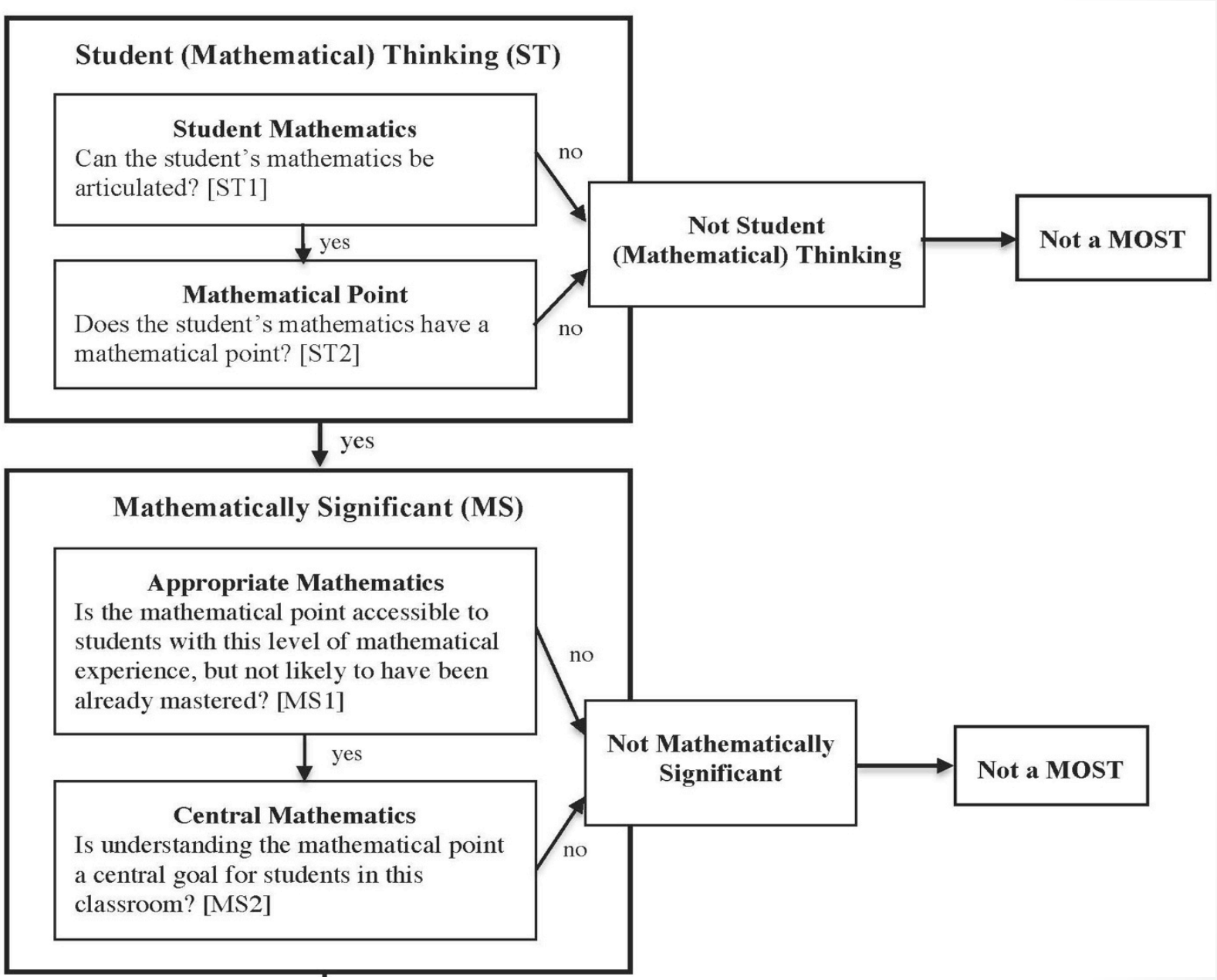
Mathematically Significant

- Must warrant use of limited instructional time
- Used in the context of teachers engaging a particular group of students in the learning of mathematics
- Two criteria that must be met
  - Appropriate Mathematics
  - Central Mathematics

Criteria for Mathematically Significant

1. Appropriate Mathematics
   - Accessible to students given their prior mathematical experiences
   - Not yet mastered
2. Central Mathematics
   - Understanding the mathematical point must be a central goal for this group of students
   - The goal could be central either to the lesson or to the discipline of mathematics


Appropriate Mathematics
Is the mathematical point accessible to students with this level of mathematical experience, but not likely to have been already mastered?
no
yes
Central Mathematics
Is understanding the mathematical point a central goal for students in this classroom?
no
yes
Does Not Satisfy Mathematically Significant Characteristic
Satisfies Mathematically Significant Characteristic


Student (Mathematical) Thinking (ST)
Student Mathematics
Can the student's mathematics be articulated? [ST1]
no
yes
Mathematical Point
Does the student's mathematics have a mathematical point? [ST2]
no
Not Student (Mathematical) Thinking
Not a MOST
yes
Mathematically Significant (MS)
Appropriate Mathematics
Is the mathematical point accessible to students with this level of mathematical experience, but not likely to have been already mastered? [MS1]
no
yes
Central Mathematics
Is understanding the mathematical point a central goal for students in this classroom? [MS2]
no
Not Mathematically Significant
Not a MOST

3 Characteristics

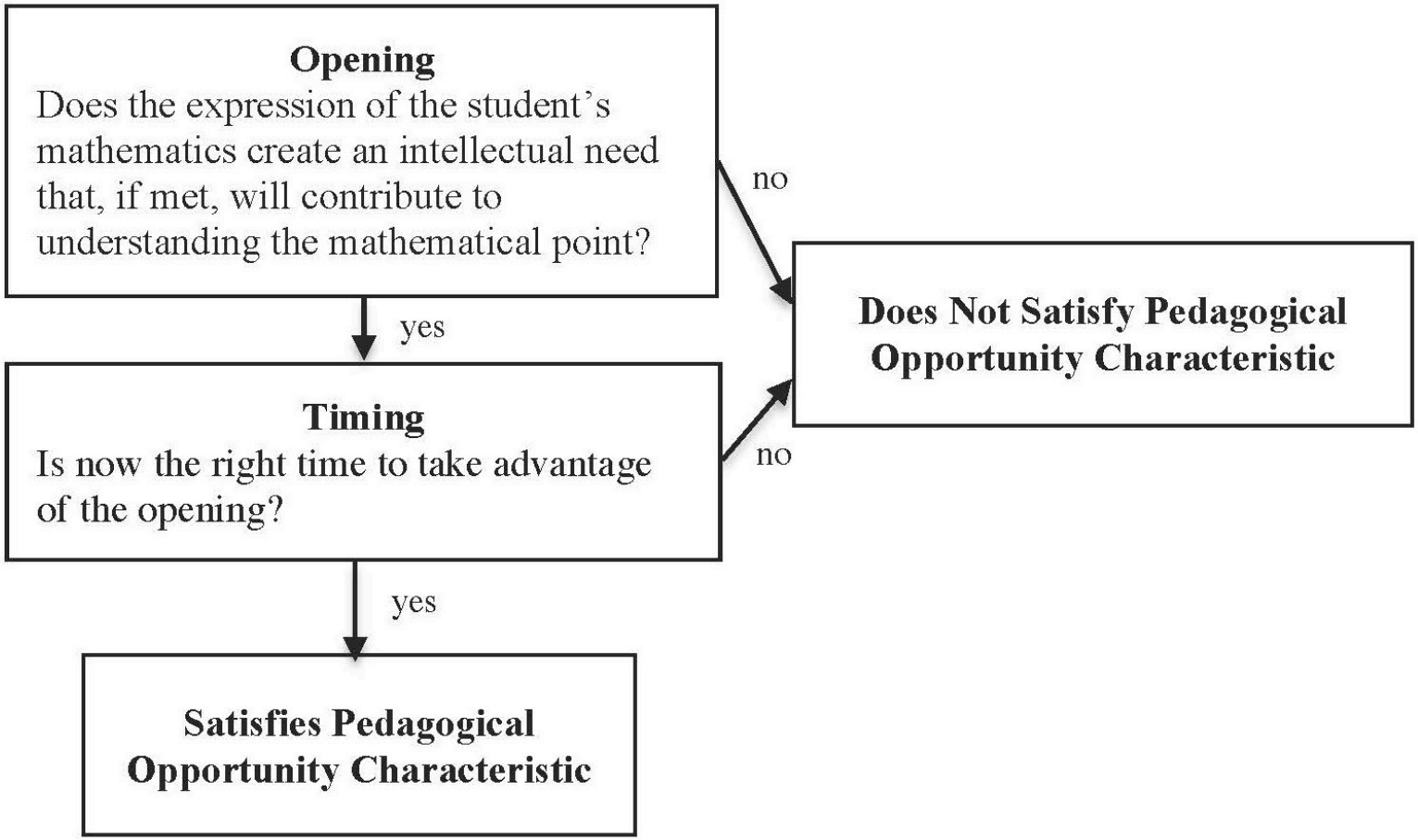
- Student Thinking
- Mathematically Significant
- **Pedagogical Opportunity**

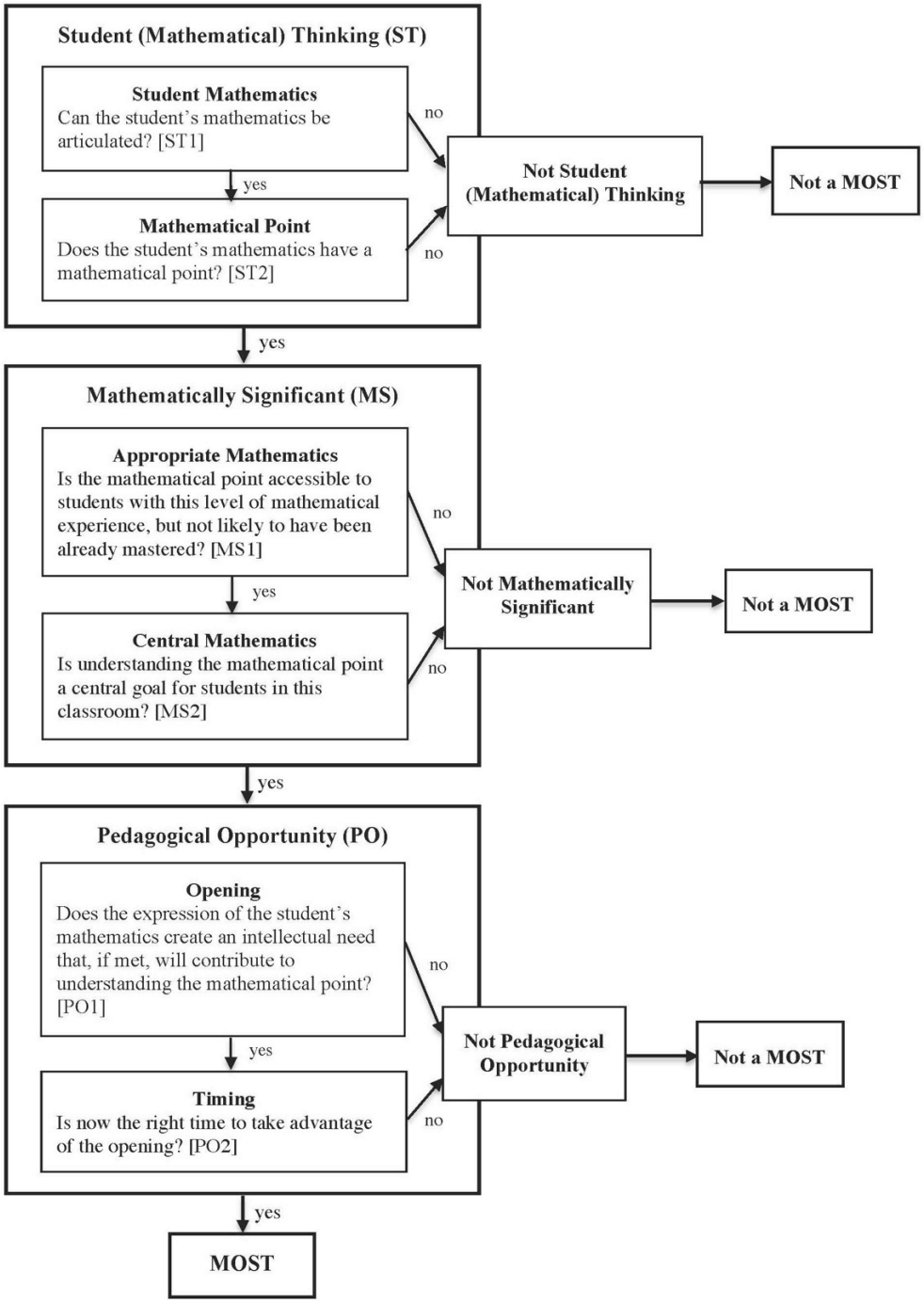
Pedagogical Opportunity
(to build on student thinking)

- An observable student action that creates an *intellectual need* (Harel, 2013) that can be acted on in that moment to contribute to students' understanding of a mathematical point.
- Two criteria that must be met
  - Opening
  - Timing

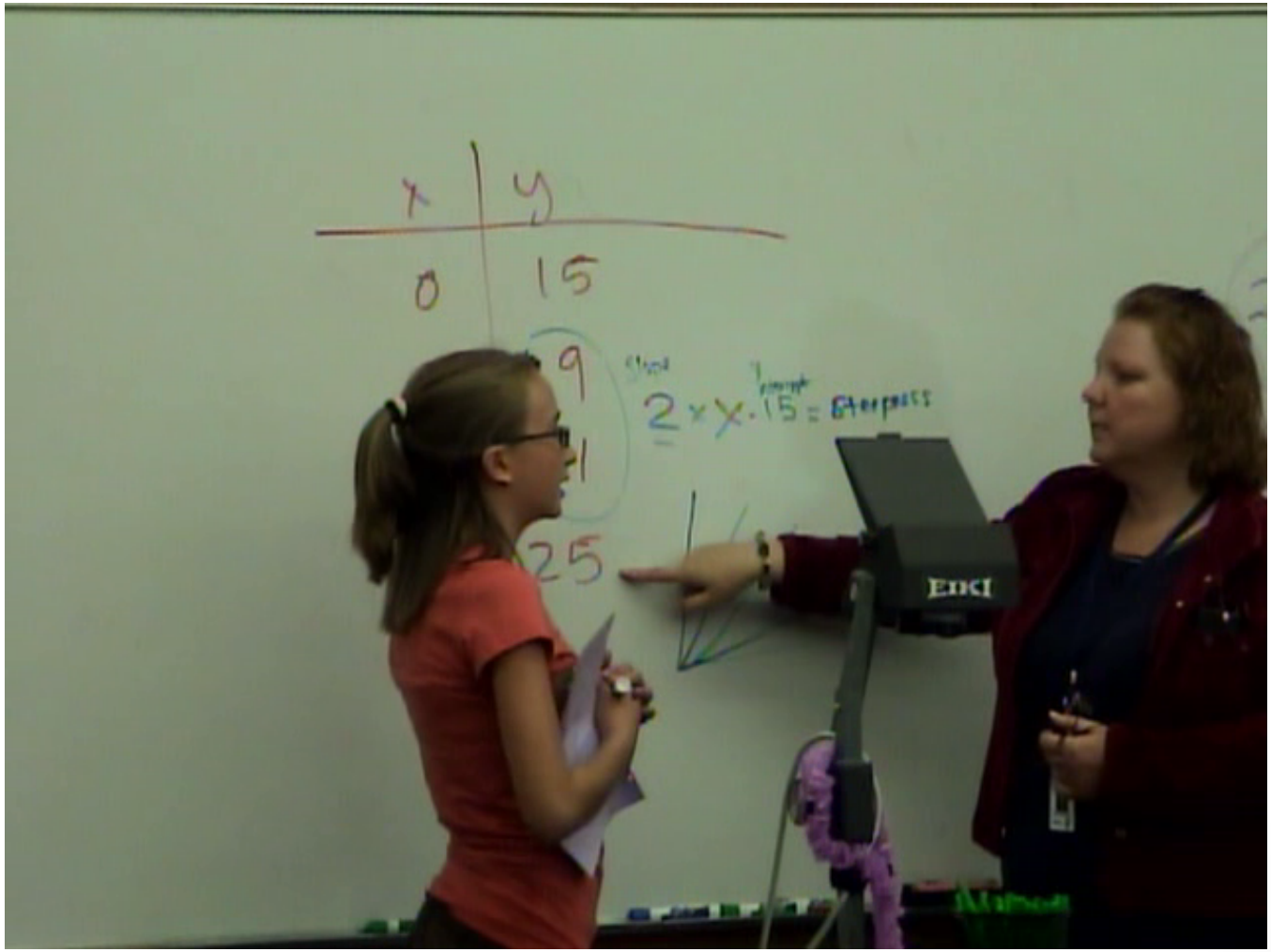
Criteria for Pedagogical Opportunity

1. Opening
   - an instance in which the expression of a student's mathematical thinking creates, or has the potential to create, an intellectual need for students to make sense of the *student mathematics*, thus providing an opportunity to understand the *mathematical point*
2. Timing
   - an opportune time to capitalize on the opening to help students understand the mathematical point of the instance


Opening
Does the expression of the student's mathematics create an intellectual need that, if met, will contribute to understanding the mathematical point?
no
yes
Timing
Is now the right time to take advantage of the opening?
no
yes
Does Not Satisfy Pedagogical Opportunity Characteristic
Satisfies Pedagogical Opportunity Characteristic


Student (Mathematical) Thinking (ST)
Student Mathematics
Can the student's mathematics be articulated? [ST1]
no
yes
Mathematical Point
Does the student's mathematics have a mathematical point? [ST2]
no
Not Student (Mathematical) Thinking
Not a MOST
yes
Mathematically Significant (MS)
Appropriate Mathematics
Is the mathematical point accessible to students with this level of mathematical experience, but not likely to have been already mastered? [MS1]
no
yes
Central Mathematics
Is understanding the mathematical point a central goal for students in this classroom? [MS2]
no
Not Mathematically Significant
Not a MOST
yes
Pedagogical Opportunity (PO)
Opening
Does the expression of the student's mathematics create an intellectual need that, if met, will contribute to understanding the mathematical point? [PO1]
no
yes
Timing
Is now the right time to take advantage of the opening? [PO2]
no
Not Pedagogical Opportunity
Not a MOST
yes
MOST

Videoclip



	Teacher (T)/Student (S)	Student Mathematics (SM)	Mathematical Point (MP)	ST	MS	PO
00:01	T: It is at a constant rate. So if I take this x and multiply it by 2 and I had 15, say x is 5, what am I going to get out? [pointing to the elements of the equation as she says them]					
00:10	S: Um, 25.	If x=5 then 2x+15 gives 25.	To evaluate a linear expression for a given value of x, the value is substituted for the x variable and appropriate operations are performed.		✓	
00:12	T: Okay. Where is 25 in our table?					
00:13	S: 5.	25 corresponds with 5 in our table.	How to identify corresponding values in a table.	✓		
00:14	T: 25. Where is it?					
00:16	S: Right there. [points to the 25 in the table]	25 is right there. [points to the 25 in the table]	None. [Simply finding the 25 on the table]	[1]		
00:17	T: What variable is that?					
00:18	S: Y.	None. [just reading what the teacher is pointing to]				
00:20	T: Okay. So now think back to what Zach said. This isn't steepness. What does this give us? [encircles "steepness" from the equation with her fingers] Back to the problem we did. [erases "steepness"]					
00:28	S: The y-variable.	The equation gives us the y-variable.				
00:28	T: Uh huh. And then in this problem, what was it?					
00:32	S: Wait, in this problem or that problem?	None.				
00:33	T: Uh huh, this problem right here that we did. What did it represent?					
00:37	S: Wait, what did what represent?	None. [clarification question]				
00:40	T: What did this column [points to y] represent in this problem that we just did?					
00:42	S: The amount saved.	The y-variable represents the amount saved.	How to identify the dependent variable from a context.			
00:43	T: Right. So that's what Zach was saying is that, this situation, it's the amount of money saved [encircles the blank space in the equation where "steepness" used to be], but since we didn't label either one of these, it's our…?					
00:50	S: Y-variable. [student then writes "y variable" in the space where "steepness" used to be]	The column with the "y" at the top represents the "y-variable."	The heading at the top of each column in a table tells what variable the entries in the column represent.			
00:51	T: Did you guys understand what she did?					
00:54	Chorus: "Yeah," "kind of," "no," and "not at all"	Cannot articulate. [referent too vague]				
01:00	T: Okay, listen. Shhhh. Listen.					
01:01	S: Okay, it is the amount between each number in the y-column [points to the y column], times the x-variable [points to the x column], plus any down payment [points to the 15 in the table] equals the y-variable.	The equation for a line if you are given any table is the amount between each number in the y-column times the x-variable plus the down payment equals the y variable. [using the points from the table (2, 19) and (3, 21) to calculate slope and (0, 15) to find the y-intercept to get 2x+15=y]				

Discussion Questions

1. What aspects of classroom mathematics discourse does this tool foreground and background?
2. What are the affordances and constraints of foregrounding and backgrounding these aspects of classroom mathematics discourse?
3. How and under what conditions could this tool be used by mathematics educators in their work, both as researchers and as mathematics teacher educators?