Mathematically Important Pedagogical Opportunities Working Group DAY 1

Laura Van Zoest (Western Michigan University)
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Related Ideas in the Literature

- Jaworski (1994) refers to "critical moments in the classroom when students created a moment of choice or opportunity" (p. 527)
- Davies and Walker (2005) use the term "significant mathematical instances" (p. 275)
- Davis (1997) calls them "potentially powerful learning opportunities" (p. 360)
- Schoenfeld (2008) refers to such moments as "the fodder for a content-related conversation" (p. 57), as "an issue that the teacher judges to be a candidate for classroom discussion" (p. 65) and as the "grist for later discussion or reflection" (p. 70)
- Schifter (1996) spoke of "novel student idea[s] that prompt teachers to reflect on and rethink their instruction" (p. 130).

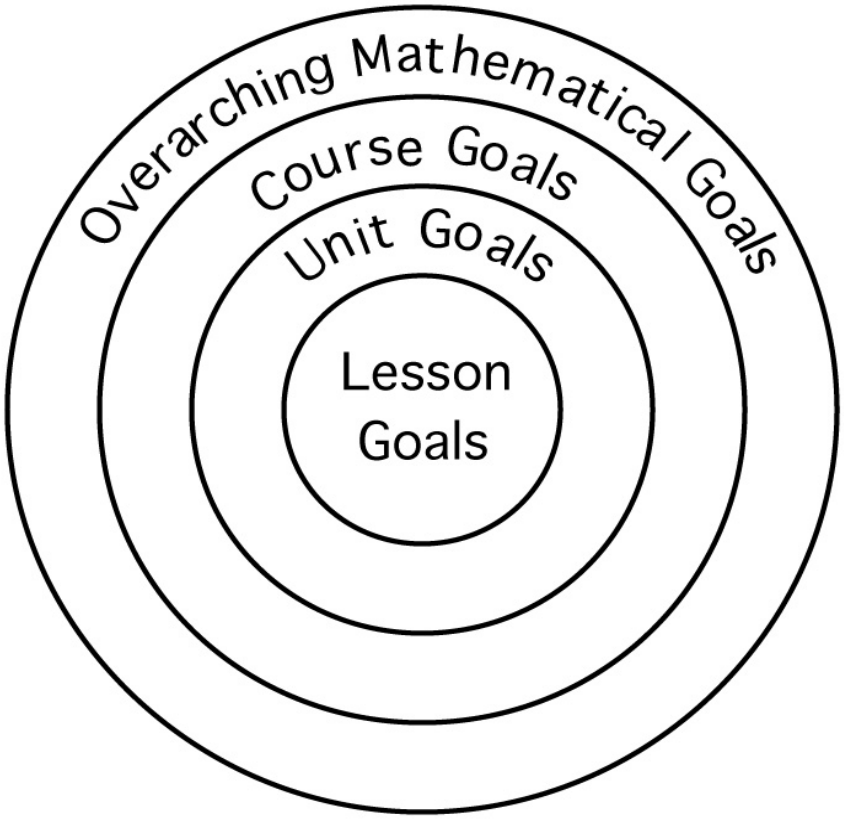
Defining MIPOs

- We define a Mathematically Important Pedagogical Opportunity (MIPO) as an *instance in a classroom lesson in which the teacher has an opportunity to extend or change the nature of students' mathematical understanding*.
- To be considered a MIPO, an instance needs to meet two important criteria:
  - it needs to involve *important mathematics*
  - it needs to be a *pedagogical opportunity*

Mathematically Important

- To be *mathematically important* in a given classroom, the instance must be centered around an idea that is related to mathematical goals for student learning.
  - In the narrowest sense, this would be a mathematical goal for the lesson in which the instance occurs
  - More broadly, the instance could also be related to the goals for a unit of instruction, an entire course, or for understanding mathematics as a whole

Layers of Mathematical Goals to which a MIPO may Relate



Mathematical Importance is Context-specific

- The mathematical importance of an instance is relative to the context in which it occurs.
- Inverse relationship between the distance of the underlying mathematical idea in the instance from the goals of the day's lesson and the needed power of the mathematical idea for it to meet the mathematically important requirement of a MIPO.
- Mathematics that may be important in one lesson or classroom may not be in another.

Pedagogical Opportunity

- Pedagogical opportunities are observable student actions that provide an opening for working towards an instructional goal.
- Pedagogical *opportunities* can be cultivated by the teacher, but cannot be created independently of the students.
- Well-executed pedagogical moves can increase the likelihood of pedagogical opportunities in a teacher's class, but the opportunities themselves come from the students, not the teacher.

MIPOs

- MIPOs occur at the intersection of *important mathematics* and *pedagogical opportunities*.
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Observable student thinking **underlies the MIPO construct.**

Classroom Episode

- Middle school pre-algebra
- Student teacher's second lesson
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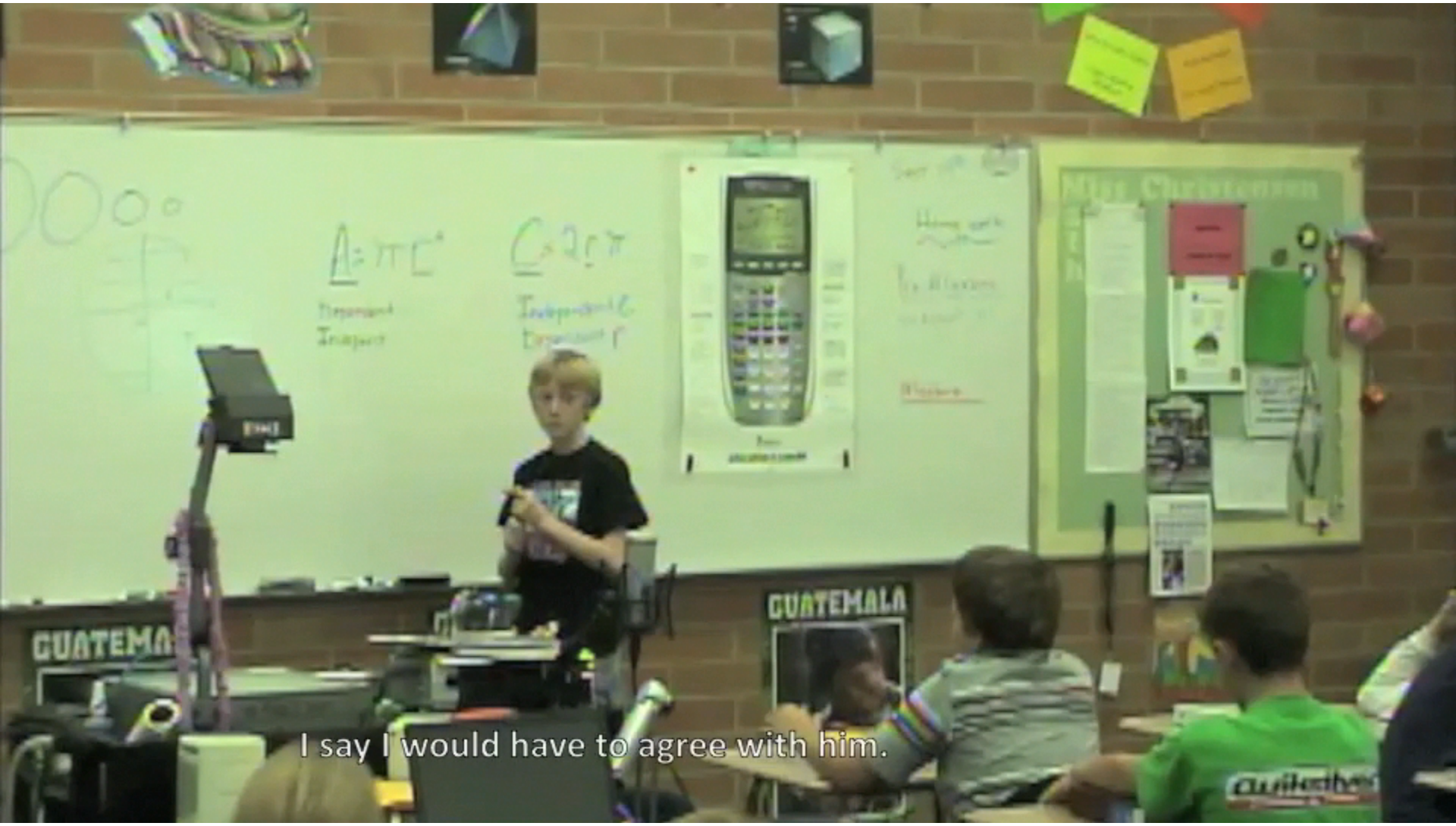
I say I would have to agree with him.

Video Discussion—Small Groups

- What MIPOs did you see?
- What made each one a MIPO?
  - What made each MIPO mathematically important?
  - What made each MIPO a pedagogical opportunity?

General Definitions

- How does one know "mathematically important" when one sees it?
- How does one know a "pedagogical opportunity" when one sees one?

Relationship among Important Mathematics, Pedagogical Opportunities and Student Thinking

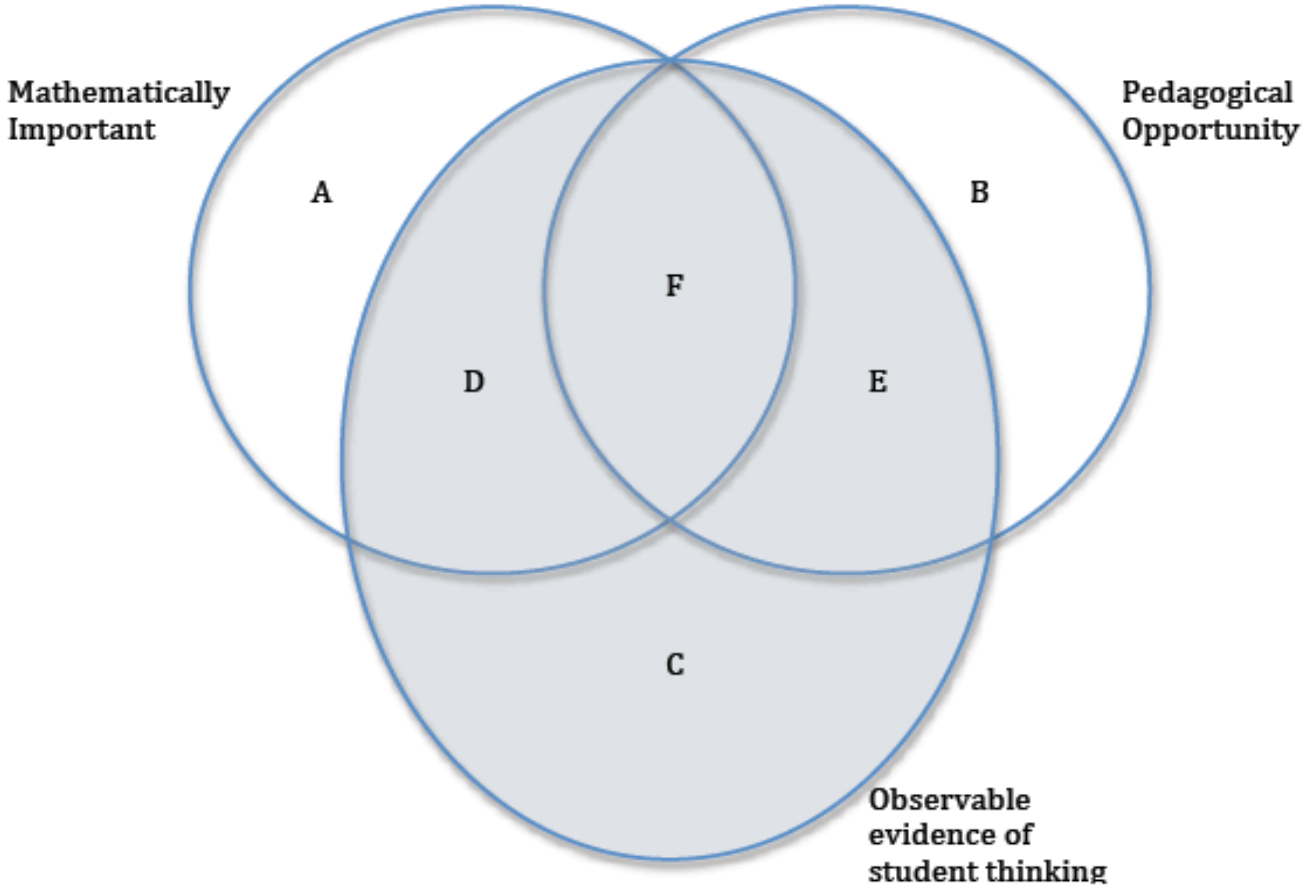

Diagram Discussion

- Think of examples of classroom instances that would fall into each region of the diagram.
- Write each example on a post-it note and place it in the appropriate region of the diagram.

Mathematically Important Pedagogical Opportunities Working Group DAY 2

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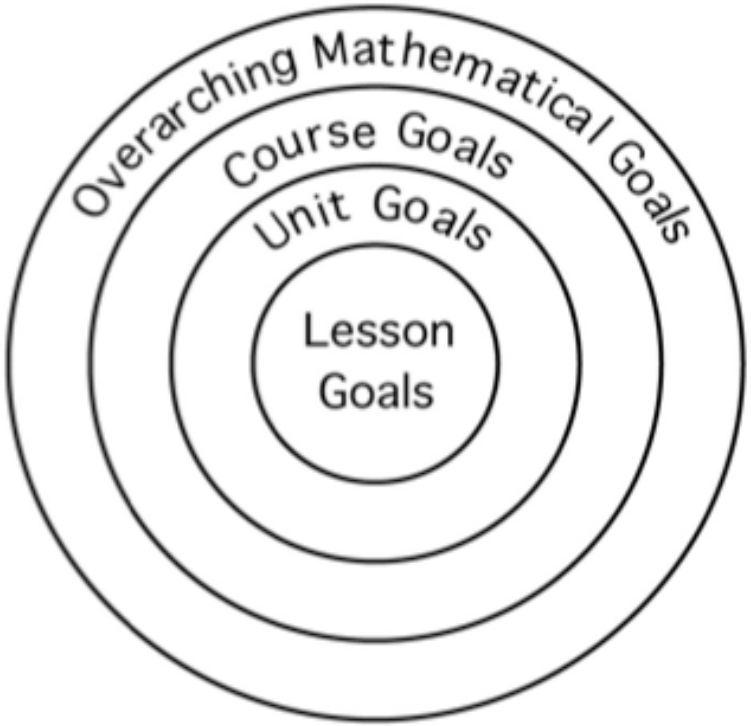

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Is There More Than This Intersection?

- Is it a 2-circle Venn diagram or a 3-circle diagram that creates this intersection?
- If student thinking underlies all MIPOs, where does it fit in a Venn diagram?
- Is all (mathematical) student thinking a MIPO?
- Is there important mathematics that doesn't involve student thinking?
- Are there pedagogical opportunities that are not mathematical or do not involve student thinking?

- Attempting to answer these questions led us to the following diagram.

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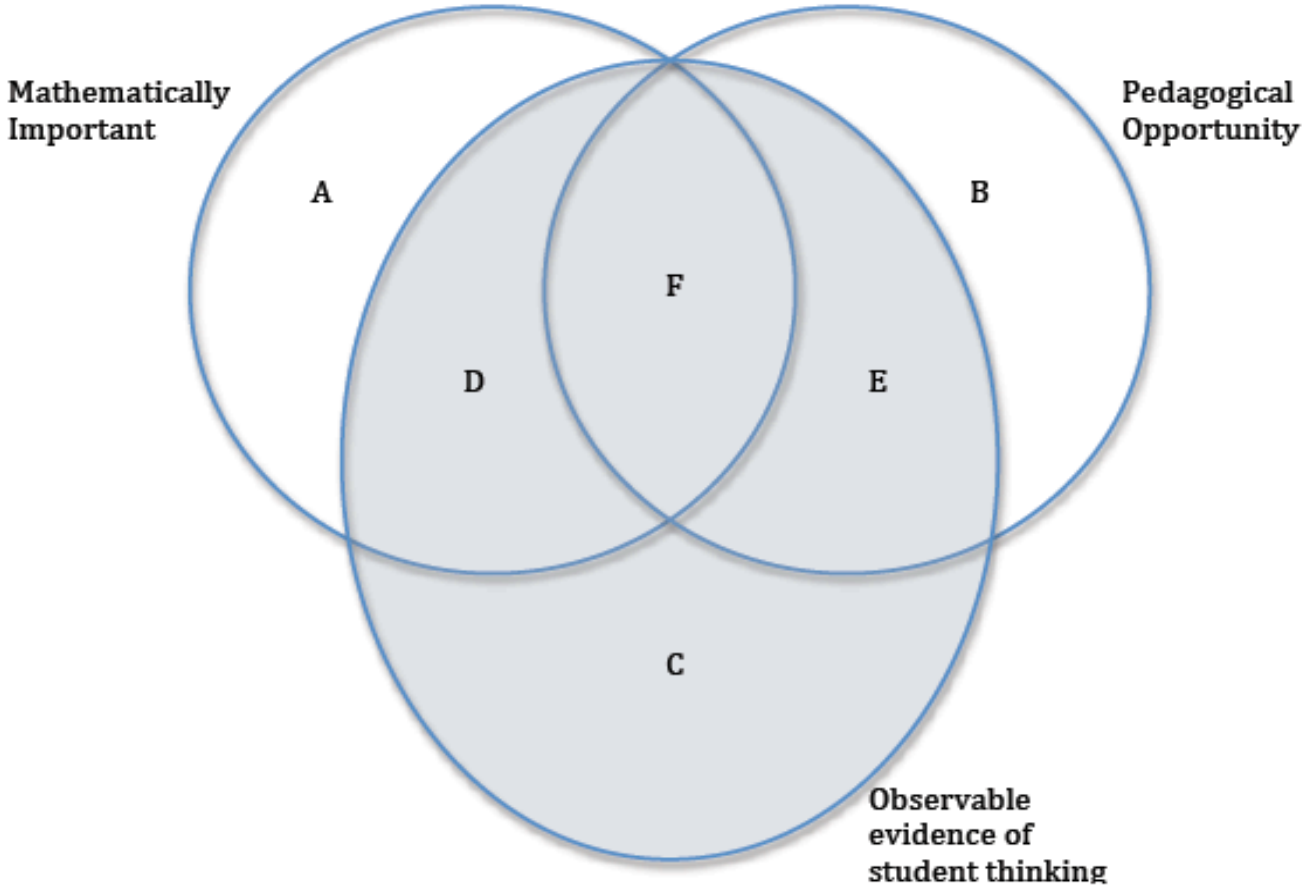
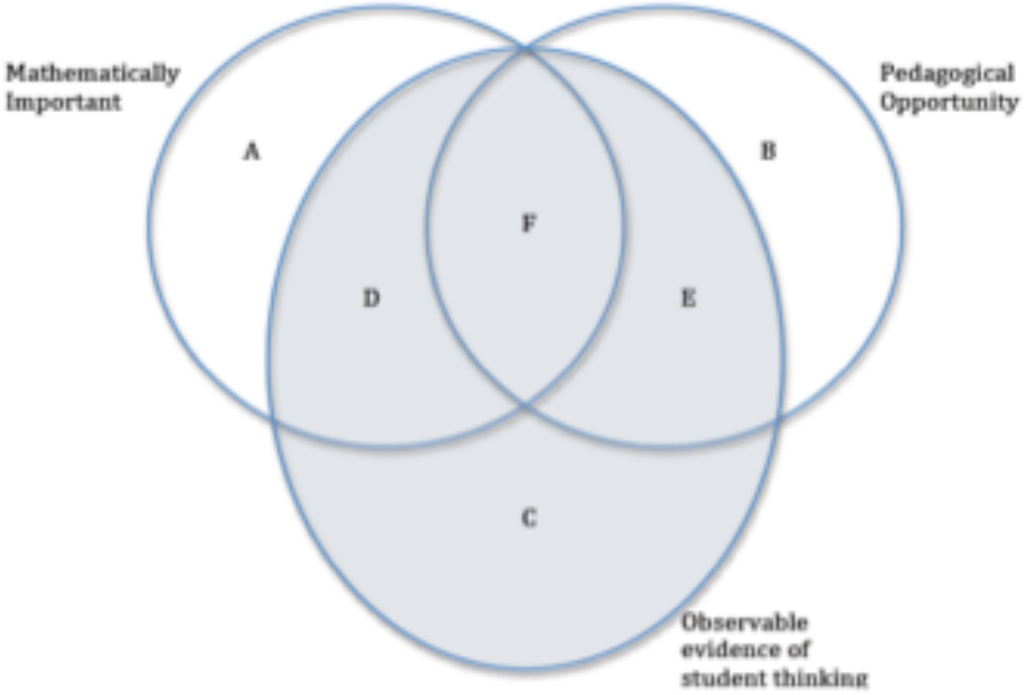


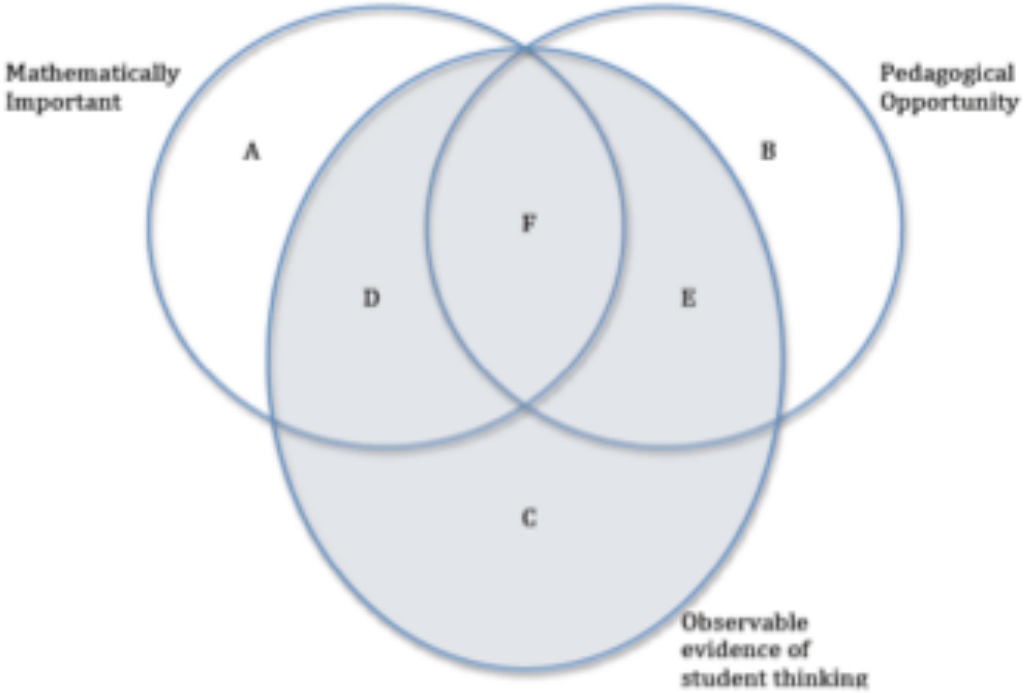
Region A

- This region represents situations that are mathematically important, but neither provide evidence of student thinking nor a pedagogical opportunity. A teacher presenting important mathematical information would fall into this region.



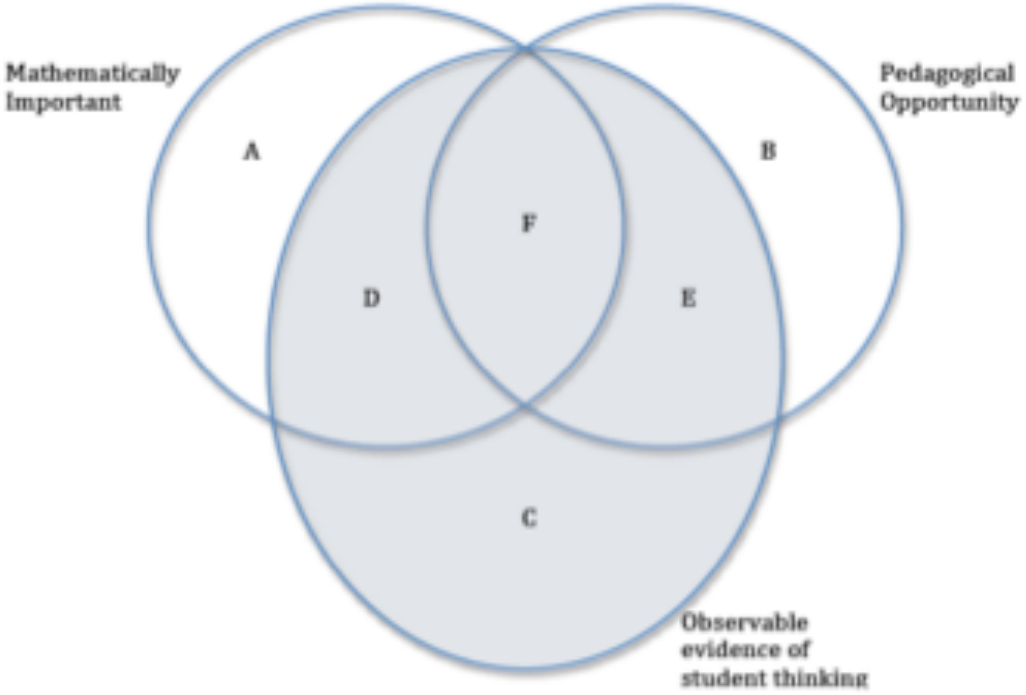
Region B

- In this region, student actions do not provide evidence of student thinking and are not mathematically important, yet provide inroads for important pedagogical goals. For example, a student were to get up to sharpen his pencil in the middle of a class discussion.



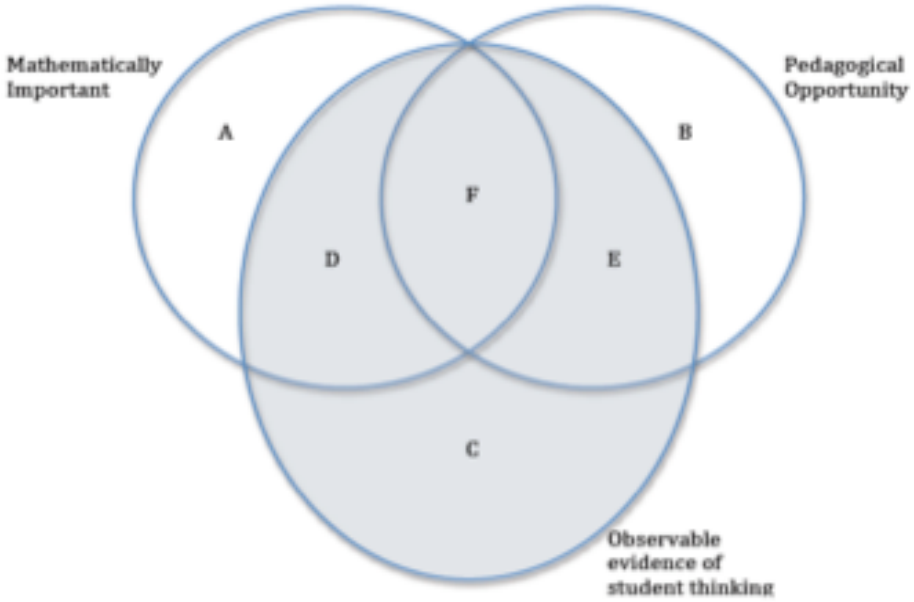
Region C

- This region represents student actions that provide evidence of their thinking, but the thinking is neither about important mathematics nor related to instructional goals. For example, a student comments about the length of the homework or reiterates a memorized fact.



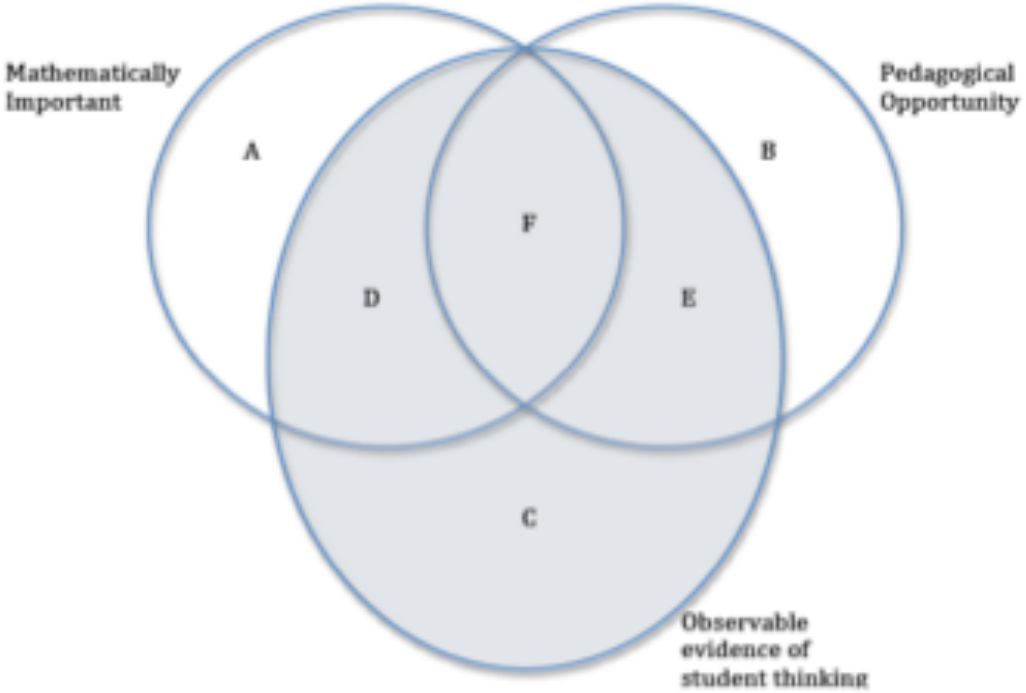
Region D

- This region represents situations of important math and evidence of student thinking, but do not provide an opening toward an instructional goal. For example, a student in an algebra class could clearly summarize why adding a constant to a linear equation would correspond to a vertical shift of the graph.



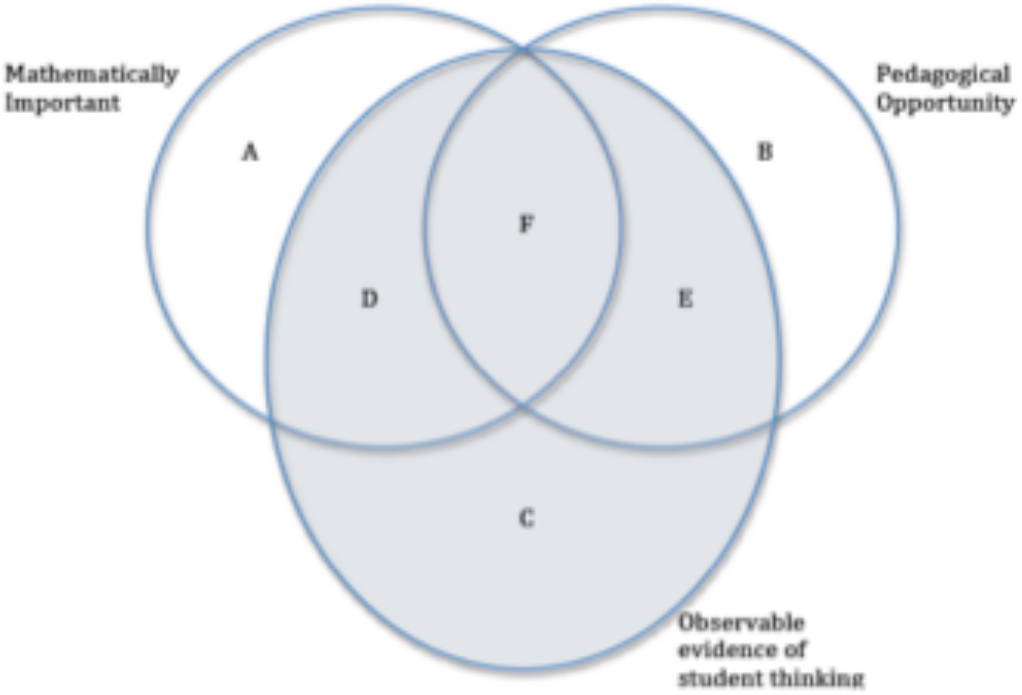
Region E

- This region represents pedagogical opportunities that provide insight into student thinking, but are not related to important mathematics. For example, a student might say, "I don't see why I need to think by myself for one minute before I talk with my group."



Region F

- Region F represents situations in which student thinking about an important mathematical idea provides an opening for working towards a mathematical goal for student learning. This is what creates a MIPO.



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- This is followed by asking students to identify the independent and dependent variables in these formulas.


GUATEMA
MALA

Mathematically Important Pedagogical Opportunities Working Group DAY 3

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Defining MIPOs

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Setting a Research Agenda

- What questions need to be addressed?
  - What knowledge or dispositions might teachers need to capitalize on MIPOs?
- What methodologies might be appropriate?
- What contexts should be considered?

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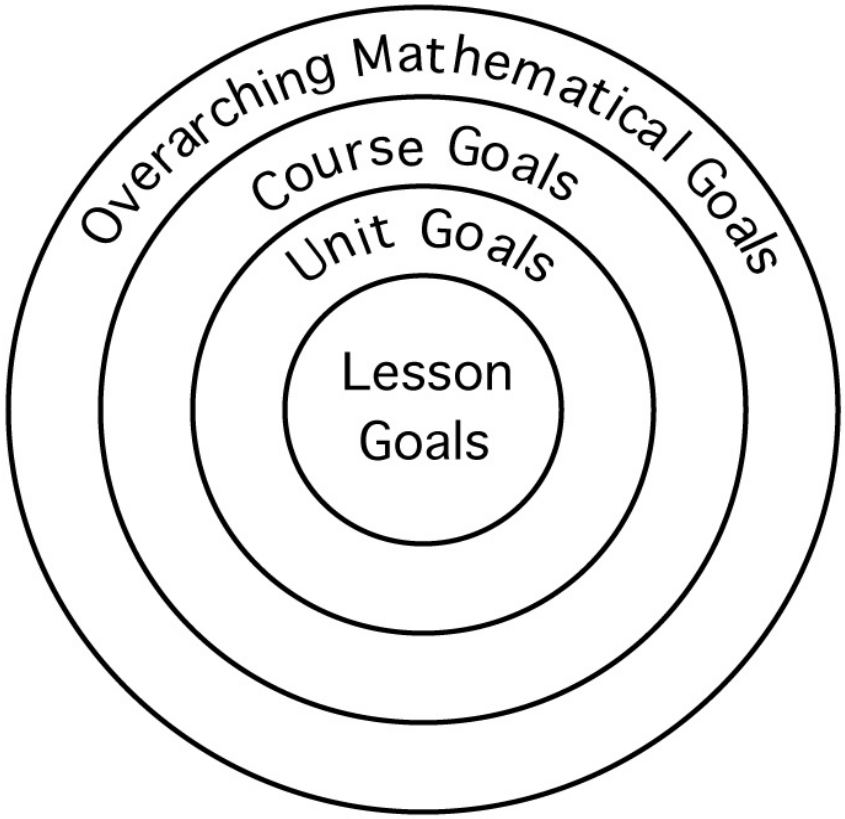
Layers of mathematical goals to which a MIPO may relate



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