INVESTIGATING MATHEMATICALLY IMPORTANT PEDAGOGICAL OPPORTUNITIES

Keith R. Leatham
kleatham@mathed.byu.edu

Blake E. Peterson
peterson@mathed.byu.edu
Brigham Young University

Shari L. Stockero
Michigan Technological University
stockero@mtu.edu

Laura R. Van Zoest
Western Michigan University
laura.vanzoest@wmich.edu

Mathematically Important Pedagogical Opportunities (MIPOs) are instances in a classroom lesson in which the teacher has an opportunity to move the class forward in their development of significant mathematics. Although this construct is widely recognized in the literature as important to mathematics teaching and learning, it is neither well defined nor clearly identified as a construct that can be studied. This working group will build on the efforts of two research groups, represented by the organizers, to define, identify, and characterize MIPOs. Specifically, Session 1 will focus on identifying MIPOs, including questioning and critiquing working definitions and preliminary dimensions of MIPOs. Session 2 will explore sub-constructs of MIPOs and the potential of sub-constructs to provide leverage in studying the broader construct. The first two sessions will include examining instances of classroom practice (written/video) that have been identified as containing MIPOs. Session 3 will focus on issues around developing a research agenda for investigating MIPOs and generating plans for continuing work on MIPOs.

History of Working Group

We propose a new working group focused on investigating mathematically important pedagogical opportunities. The two research groups represented by the organizers have been interacting over the past year, including meeting informally at PME-NA 2009, around their overlapping work on defining, identifying, and characterizing mathematically important pedagogical opportunities. The working group will be an opportunity to expand this work to include a broader range of researchers interested in this topic.

Focal Issues

Although skilled teachers and teacher educators often intuitively “know” when mathematically important pedagogical opportunities occur during a lesson and can readily produce ideas about how to capitalize on such opportunities, the literature reveals a construct that is not well-defined. Ideas related to these opportunities are mentioned in many different ways. For example, Jaworski (1994) refers to such opportunities as “critical moments in the classroom when students created a moment of choice or opportunity” (p. 527). Davies and Walker (2005) use the term “significant mathematical instances” (p. 275) and Davis (1997) calls them “potentially powerful learning opportunities” (p. 360). Schoenfeld (2008) refers to such moments as “the fodder for a content-related conversation” (p. 57), as “an issue that the teacher judges to be a candidate for classroom discussion” (p. 65) and as the “grist for later discussion or reflection” (p. 70). Schifter (1996) spoke of “novel student idea[s] that prompt teachers to reflect on and rethink their instruction” (p. 130). In the context of teacher professional development, Remillard and Geist (2002) described *openings in the curriculum* as moments in which teachers’ questions, observations, or challenges require the facilitator to make a decision about how to incorporate

Brosnan, P., Erchick, D. B., & Flevares, L. (Eds.). (2010). Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Columbus, OH: The Ohio State University.

into the discussion the mathematical or pedagogical issues that are raised. The nature of the facilitator's decision determines the extent to which the teachers' ideas advance the learning of the group. Similarly, when a MIPO occurs in a classroom lesson, a teacher first needs to recognize it as such, and then make a decision about how to respond. Depending on the action taken by the teacher, the MIPO may or may not support the development of students' mathematical understanding.

Thus, there is clear recognition that such opportunities, whatever they are called, are important to mathematics teaching and learning. These opportunities, however, frequently either go unnoticed or are not acted upon by many teachers, particularly novices (Peterson & Leatham, 2010). This raises the question of how teacher educators can help teachers recognize MIPOs during their instruction and use them to support student learning. The purpose of this working group is to engage with the construct of MIPOs in order to consider ways that researchers can identify, explore, and study these opportunities.

In the following, we first give a brief overview of the work around MIPOs done by each of the two research groups. We then provide a description of how we will engage participants with the construct of MIPOs during the three working group sessions, and end by outlining potential follow-up activities.

A Broad View of Mathematically Important Pedagogical Opportunities
Leatham & Peterson

For us, a MIPO is when students' observed mathematical thinking provides the teacher with an opportunity to move the class forward in their development of significant mathematics. MIPOs are driven by significant mathematics and observed student thinking. When students articulate mathematical ideas, expert mathematics teachers use their professional knowledge to discern whether there is evidence that students are ready, at that moment, to engage with important underlying mathematics. Many teachers, however, do not recognize MIPOs in their classrooms. From our observations of novice teachers, MIPOs often go unrecognized because of the wide variation of circumstances under which they occur. We describe four types of variation in MIPOs: two related to the nature of mathematics within a MIPO (focus and problematization) and two related to the way in which a MIPO surfaces (visibility and predictability). Our belief is that an awareness of these variations is a first step to helping teachers recognize MIPOs more often in their classrooms.

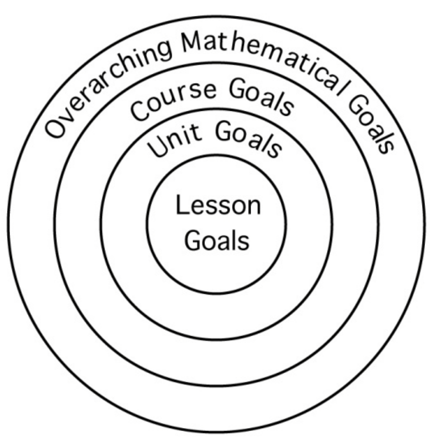

Figure 1. Layers of mathematical goals one might consider in recognizing MIPOs

The nature of the mathematics in MIPOs varies in at least two important ways: focus and problematization. First, MIPOs vary in the degree to which they support the mathematics a given lesson or discussion was designed to elicit. Thus, although MIPOs could lead to discussion about the mathematics within the content of the day's lesson, they could also lead to discussion about mathematics related to broader goals of the unit, course or mathematics as a whole (see Figure 1). To be prepared for such variation in mathematical focus, Leinhardt and Steele (2005) suggested that teachers need "both a primary mathematical agenda… and sensible, valued, predefined classes of situations that suggest deviating from the agenda" (p. 110).

Second we consider variation with respect to problematization. For us MIPOs are always an opportunity to help students to clarify important mathematical ideas. There is variation, however, in the degree to which students initially see the need for this clarification—that is, already see the mathematics of the situation as problematic. On one extreme, students may raise the problematic nature of the mathematics themselves (e.g., "But yesterday you said…"). On the other extreme students may share their thinking, yet not be aware, as far as the teacher has evidence, that there is something problematic or noteworthy in the mathematics at hand. In such situations teachers have the opportunity to help students to problematize the situation.

In addition to variation related to mathematics, the third and fourth dimensions relate to the ways in which teachers observe students' mathematical thinking in a MIPO: visibility and predictability. The third dimension is the variation in how MIPOs are made visible to teachers. On one extreme, students may simply share their thinking with little teacher elicitation (e.g., student raises their hand and asks a question). On the other hand, the teacher may have observed students' mathematical thinking as they worked in small groups or individually. In these instances, the teacher may choose to share the thinking (e.g., "I saw several students using this strategy…") or ask the student to do so. Thus MIPOs initially may become visible to the teacher in the context of whole class, small group, or individual work, and may or may not be made public by the students themselves.

Finally, MIPOs vary in the degree to which they are predictable. For example, teachers often engage students in tasks that are meant to elicit particular mathematical thinking; thus teachers often anticipate certain solution strategies and the mathematics to which those strategies give rise. In addition, over time teachers begin to recognize common student conceptions and misconceptions. Because of this anticipation and knowledge of students' thinking, opportunities often arise that are predictable. Even when the thinking is predictable, however, the details often are not. For example, we may anticipate *that* students will respond in certain ways, but just *when* they do so and *how* they articulate their thinking are often less predictable. Some MIPOs are even less predictable. There are a multitude of pathways down which students' thinking can take them, particularly when they are engaged in classrooms that encourage them to try to actively make sense of the mathematics at hand.

In our experience, teachers most easily recognize MIPOs with these characteristics: (1) the mathematical focus aligns well with the day's lesson, (2) there is evidence that students see the mathematics at hand as problematic, (3) the thinking is made visible in the context of a whole class discussion, or (4) the thinking was anticipated. An understanding of the variations we have just described, however, can help teachers to recognize less obvious variations of MIPOs, particularly when the mathematical focus is beyond the day's lesson, students do not yet perceive the situation as problematic, the thinking is made visible in small group or individual conversation, or the thinking is unexpected. Although we find these variations useful, they do not help one to identify when occurrences of student mathematics thinking are MIPOs. To aid in their identification, we are in the process of describing common types of MIPOs. Two common types are (1) incorrect or incomplete responses and (2) multiple responses or solutions. First, incorrect responses are often MIPOs because giving students opportunities to discuss the thinking behind their responses leads to important mathematics related both to why the response was incorrect as well as the mathematics underlying a correct response. Similarly, vague or incomplete thinking is also a potential MIPO. Encouraging students to elaborate and clarify often reveals confusion and questions related to important underlying mathematical ideas.

Another common type of MIPO occurs when there exist multiple student responses or solutions to a given question. These situations give rise to MIPOs for a couple of reasons: (1) the solutions are inconsistent or (2) there are commonalities between different methods. When solutions are inconsistent by one being correct and the other incorrect, students can evaluate the thinking that supports these solutions to identify the critical elements of the underlying concepts. If two correct solutions have relied on different methods, a comparison of these methods can highlight the underlying mathematics that is common to both. An awareness of these common types of MIPOs can help teachers learn how to identify them. We expect there are other common types worth exploring.

Pivotal Teaching Moments as a Sub-construct of MIPOs
Stockero & Van Zoest

We have narrowed our work on MIPOs to focus on a sub-construct that we call *pivotal teaching moments* (PTM). We define a PTM as *an instance in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend, or change the nature of, students' mathematical understanding*. We ground our work in classroom observations of beginning teachers' practice based on the hypothesis that the decision-making process involved with PTMs may be more obvious with this group than with skilled teachers. That is, skilled teachers may recognize a PTM and make the decision to act so quickly and smoothly that it wouldn't be clear that they had modified their instruction.

We believe that an important first step in capitalizing on PTMs is recognizing that such moments exist. Without this awareness, teachers may experience *inattentional blindness* (Simons, 2000)—a phenomenon described in the psychology literature as a failure to focus attention on unexpected events. This is also related to the idea of *framing* (Levin, Hammer & Coffey, 2009)—the way in which a teacher makes sense of a classroom situation. From this perspective, whether a teacher notices the value in an unexpected event depends on how he or she frames what is taking place during instruction. If, for example, a teacher views a student error as something that needs to be corrected, he or she is unlikely to consider the mathematical thinking behind the error or whether the error could be used to highlight a specific mathematical idea. On the other hand, a teacher who views an error as a site for learning is more likely to consider both the mathematics underlying the error and how it could be used to develop mathematical understanding. Thus, we began our research by investigating the circumstances in which PTMs seem likely to occur.

In general, PTMs seem most likely to occur when students are actively engaged in and contributing to the mathematics lesson in some way—either by the design of the lesson or by their own initiative. In our work to date, we have identified five circumstances that are fertile grounds for PTMs. These are circumstances in which: (1) student comments or questions go beyond the mathematics that the teacher had planned to discuss; (2) students are trying to make sense of the mathematics in the lesson; (3) incorrect mathematical thinking or an incorrect solution is made public; (4) a mathematical contradiction occurs; and (5) confusion is expressed about specific mathematical ideas.

Drawing on Stein, Smith, Henningsen, and Silver's (2000) work on cognitive demand of tasks, we have conceptualized PTMs as triples that include the PTM, the teacher decision and the likely impact on student learning. We further characterize the first component of the triple, the PTM, by the circumstance that led to it and the potential of the moment to improve students' opportunities to learn mathematics. The second component, teacher decision, is characterized by the action the teacher takes and how effectively they implement that action. Teacher actions that we have identified include: (1) ignoring or dismissing the interruption; (2) incorporating the interruption into the plan; (3) pursuing student thinking; (4) emphasizing the meaning of the mathematics; and (5) extending the mathematics or making connections to other topics. The final component of the triple, likely impact on student learning, is identified as negative impact, neutral impact, or low, medium, or high positive impact.

This PTM work provides an instance of studying a sub-construct of MIPOs that can be used to inform the working group's thinking about developing frameworks and methodologies for studying the broader MIPO construct.

Plan for Working Group

Session 1: Identifying mathematically important pedagogical opportunities

Session 1 will begin with introductions of the working group organizers and participants to gain a sense of participants' contexts and the reasons they are interested in working on or thinking about the MIPO construct. In order to allow participants to more easily become engaged in discussions about MIPOs, particularly those for whom the idea is new, a short presentation will be given by the organizers related to defining, characterizing, and identifying MIPOs. This will be followed by small and large group discussions that will provide particpants an opportunity to question and critique the ideas that have been presented. The group discussions will center on questions and ideas about MIPOs with which the organizers have been wrestling, including:

- How does one know "mathematically important" when one sees it?
- To what extent does the working definition capture the idea of a MIPO? What is not being captured by this definition?
- Do the dimensions described in the presentation help one think about MIPOs? Are there other dimensions that should be considered? What is the relative importance of these dimensions?
- Can MIPOs be both planned and unplanned?

Classroom video or written clip(s) that have been identified as containing MIPOs will then be shared both as a means of further engaging participants with the idea of a MIPO and of introducing the types of activities in which participants will engage in Session 2. Session 1 will conclude with a short discussion related to issues that will be discussed further in Sessions 2 and 3, including how the MIPO construct might be narrowed in order to gain some traction in understanding the construct and issues of practicality and interest. In particular, Sessions 2 and 3 will be responsive to the interests of participants through the selection of the instances of classroom practice and the implementation issues on which we will focus.

The goals of Session 1 are to:

- Develop an understanding of participants' contexts and interests in MIPOs in order to structure the working group sessions in a way that is responsive to the participants.

- Engage participants in thinking about the idea of a MIPO by presenting an example of how they have been defined, characterized, and identified.
- Begin to refine definitions and ideas related to MIPOs through small and whole group discussions.

Session 2: *Exploring mathematically important pedagogical opportunities*

To stimulate a conversation about the possibility of sub-constructs that could be used to gain leverage in making sense of the broader MIPO construct, Session 2 will begin by introducing pivotal teaching moments (PTMs) as an example of a sub-construct. The PTM work was chosen because it provides an example of how narrowing the scope of the MIPOs can lead to the development of frameworks and methodologies that may be applicable to studying the broader MIPO construct.

After giving a brief overview of work on PTMs, we will engage working group members in examining instances of practice (written/video) that have been identified as containing MIPOs. This will provide an opportunity to begin to test out and refine existing definitions and frameworks related to both PTMs and the broader MIPO construct, as well as raise additional issues about MIPOs.

The goals of Session 2 are to:

- Test out and refine existing MIPO definitions and frameworks.
- Consider sub-constructs of MIPOs that might provide leverage in studying the broader construct.

Session 3: Setting a research agenda for investigating mathematically important pedagogical opportunities

Session 3 will focus on issues around developing a research agenda for investigating MIPOs. We will begin by considering what the field needs to know about MIPOs in order to use the construct to improve the practice of both mathematics teacher educators and classroom teachers. For example, what knowledge or dispositions might teachers need to capitalize on MIPOs? Specifically, we will discuss:

- What questions need to be addressed?
- What methodologies might be appropriate?
- What contexts should be considered?

Based on these discussions, small groups will be formed around the working group participants' expressed interests. Collaborative endeavors will be encouraged and venues for continuing MIPO work will be discussed.

The goals of Session 3 are to:

- Outline a research agenda for studying MIPOs.
- Generate plans for continuing MIPO work.

Anticipated Follow-up Activities

The work of the group will be continued by the core research teams through ongoing collaboration. If there is sufficient interest among working group participants, the organizers will facilitate broader research collaborations and additional working group sessions through venues such as PME-NA conferences, AMTE pre-sessions, and web-conferencing.

References

Davies, N., & Walker, K. (2005). Learning to notice: One aspect of teachers' content knowledge in the numeracy classrooms. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, Pierce, R. & Roche, A. (Eds.), *Building connections: Theory, research and practice. In Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia* (pp. 273-280). Sydney, Australia: MERGA.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education, 28*, 355-376.

Jaworski, B. (1994). *Investigating mathematics teaching: A Constructivist Enquiry*. London: Falmer Press.

Leinhardt, G. & Steele, M.D. (2005). Seeing the complexity of standing to the side: Instructional dialogues. *Cognition and Instruction, 23,* 87-163.

Levin, D. M., Hammer, D., & Coffey, J. E. (2009). Novice teachers' attention to student thinking. *Journal of Teacher Education, 60*(2), 142-154.

Peterson, B. E., & Leatham, K. R. (2010). Learning to use students' mathematical thinking. In L. Knott (Ed.), *The role of mathematics discourse in producing leaders of discourse* (pp. 99-128). Charlotte, NC: Information Age.

Remillard, J. T., & Geist, P. K. (2002). Supporting teachers' professional learning by navigating openings in the curriculum. *Journal of Mathematics Teacher Education, 5,* 7-34.

Schifter, D. (1996). *What's happening in math class?* New York, NY: Teachers College.

Schoenfeld, A. H. (2008). On modeling teachers' in-the-moment decision making. In A. H. Schoenfeld (Ed.), *A study of teaching: Multiple lenses, multiple views, JRME monograph #14* (pp. 45-96). Reston, VA: National Council of Teachers of Mathematics.

Simons, D. J. (2000). Attentional capture and inattentional blindness. *Trends in Cognitive Science, 4*(4), 147-155.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.