Imprecision in Classroom Mathematics Discourse

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Communication is Problematic

Example A

The proportion $\frac{6}{4} = \frac{x}{10}$ is on the board.

Teacher: How could you solve this proportion for x?

Student: By multiplying it by 10.

Example B

The proportion $\frac{6}{4} = \frac{x}{10}$ is on the board.

Teacher: How could you solve this proportion for x?

Student: By dividing.

IMPRECISION

If a part of speech is used such that

- it can be interpreted in multiple viable ways,
- and the existence of those interpretations causes the overall meaning of the statement in which it occurs to be unclear,

then we refer to both the part of speech and the statement in which it occurs as *imprecise*.

Identifying imprecision

T=Teacher ***S=Student***		**Imprecision Type**
The proportion $\frac{6}{4}=\frac{x}{10}$ is on the board. T: How could you solve this proportion for x?		
A	**S: By multiplying it by 10.**	Subject- pronoun
B	**S: By dividing.** them. Divide *what* by *what*?	Object of verb- implied
Given $P=2.50V-500$, $V=600-500R$ the class was asked to (1) solve for P given $R=.25$ (2) solve for R given $P=625$ The class is now comparing the work involved in solving (1) and (2).		
C	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.** The opposite of *what*?	Object of adjective- implied
D	**S: In (1) you just do the equation instead of doing multiple step equations.**	Verb

Imprecision

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Addressing Imprecision

Honing In

Addressing imprecision

T=Teacher ***S=Student***		**Possible Teacher Response**
The proportion $\frac{6}{4}=\frac{x}{10}$ is on the board. T: How could you solve this proportion for x?		
A	**S: By multiplying it by 10.**	When you say "multiply *it* by 10", what is the *it* that's being multiplied?
B	**S: By dividing.**	
Given $\begin{array}{l} P=2.50V-500 \\ V=600-500R \end{array}$ the class was asked to (1) solve for P given $R=.25$ (2) solve for R given $P=625$ The class is now comparing the work involved in solving (1) and (2).		
C	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.**	So, for (2) you had to "do the opposite" of something. What was that *something*?
D	**S: In (1) you just do the equation instead of doing multiple step equations.**	

Imprecision

If a **part of speech** is used such that

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Addressing Imprecision: Honing in

To address imprecision:

- Identify the imprecision
- Ask the student a clarification question that
  - Establishes what you *do* understand
  - *Is specific to the imprecise part of speech*

C	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.**	So, for (2) you had to "do the opposite" of something. What was that *something*?

Why do we address imprecision this way?

- Honors all the good stuff that the student said
- Highlights the part of speech that needs to be cleared up
  - Focuses your question

Addressing imprecision

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The proportion $\frac{6}{4}=\frac{x}{10}$ is on the board. T: How could you solve this proportion for x?		
A	**S: By multiplying it by 10.**	When you say "multiply *it* by 10", what is the *it* that's being multiplied?
B	**S: By dividing.**	Divide *what* by *what*?
Given $P = 2.50V - 500$, $V = 600 - 500R$ the class was asked to (1) solve for P given $R = .25$ (2) solve for R given $P = 625$ The class is now comparing the work involved in solving (1) and (2).		
C	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.**	So, for (2) you had to "do the opposite" of something. What was that *something*?
D	**S: In (1) you just do the equation instead of doing multiple step equations.**	When you say "*do* the equation," what do you mean by *do*?

To address imprecision:

Identify the imprecision

Ask a clarification question that:

- Establishes what you *do* understand
- *Is specific to the imprecise part of speech*

Why is it important to address imprecision?

Or put another way …

What are the ramifications of not addressing an instance of imprecision?

Consider the proportion problem $\frac{6}{4} = \frac{x}{10}$, when a student claims they can solve the proportion "by multiplying it by 10"

What are the rest of the students thinking about this student's claim *if the imprecision goes unaddressed*?

- They make an inference about the meaning of the claim.
  - *Their inference must be correct*. *(Regardless of the actual accuracy)*
  - Ramification: Might think they understand when they actually don't.
- They don't know what that student means.
  - *Everyone else must understand so I don't know what is going on.*
  - Ramification: Might think they don't understand when they actually do.

Consider the problem with two equations

$$P = 2.5V - 500$$
$$V = 600 - 500R$$

What if some student interpret "do the equation" as

$$V = 600 - 500 \times .25$$

and other students interpret "do to equation" as

$$R = \frac{V - 600}{-500}$$

If the imprecision goes unaddressed, what happens in the classroom communication moving forward?

- Each group of students assumes their interpretation is correct.
- Subsequent statements are seen in light of each students' own interpretation
- Ramification: Inconsistent parallel conversations

Long-term Implications

- Students disengage from class discourse
- Missed opportunities for the teacher to understand student mathematical thinking
- Student misconceptions develop or persist

Questions & Comments

Discussion Questions

In our paper we make the following claim:

"Although it is unwise to ask for clarification about every student statement, it is equally unwise to never seek clarification."

How might the definition we have provided help one to know when to seek clarification?

How might "imprecision" be connected to the Mathematical Practice "Attend to Precision"?

Discussion Questions

How do you support teachers in attending to this type of precision?

In what ways might teachers actually contribute to imprecision in the classroom?

How can classroom norms be created to help students identify and address instances of imprecision?

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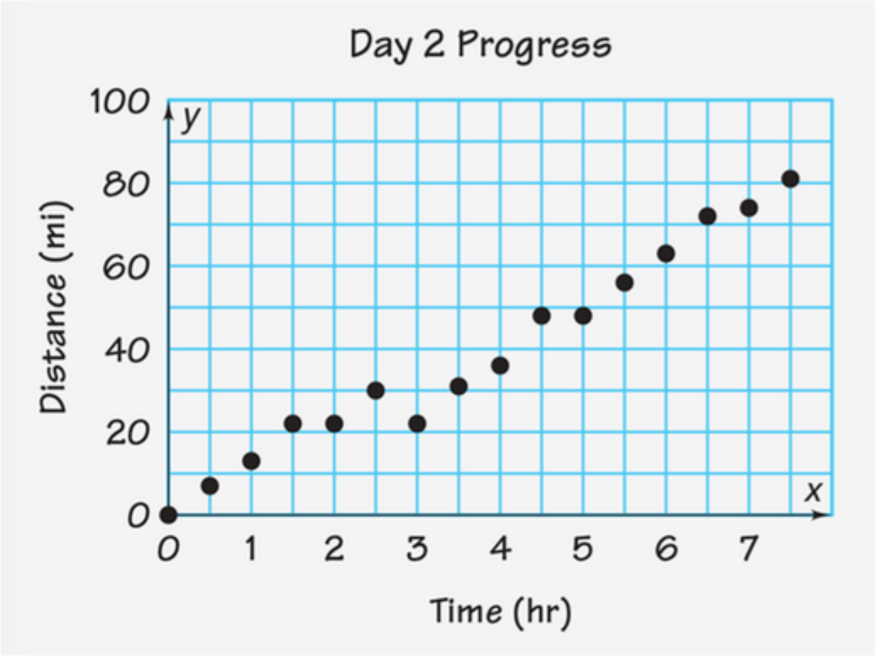
Thank you!

As a teacher,
be careful not to introduce imprecision

- Your own imprecise parts of speech
- Multiple questions



T: “So this is stopped [pointing to the interval between 1.5 and 2]. Are we all okay with that? This is stopped.”

S: “And then they went up.”

T: “So what does that mean?”

S: “They went up.”

T: “Does that mean they went up a hill? What do you mean by they went up?

S: [Student’s making hand gestures indicating positive slope.]

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