Mathematical Opportunities in Student Thinking

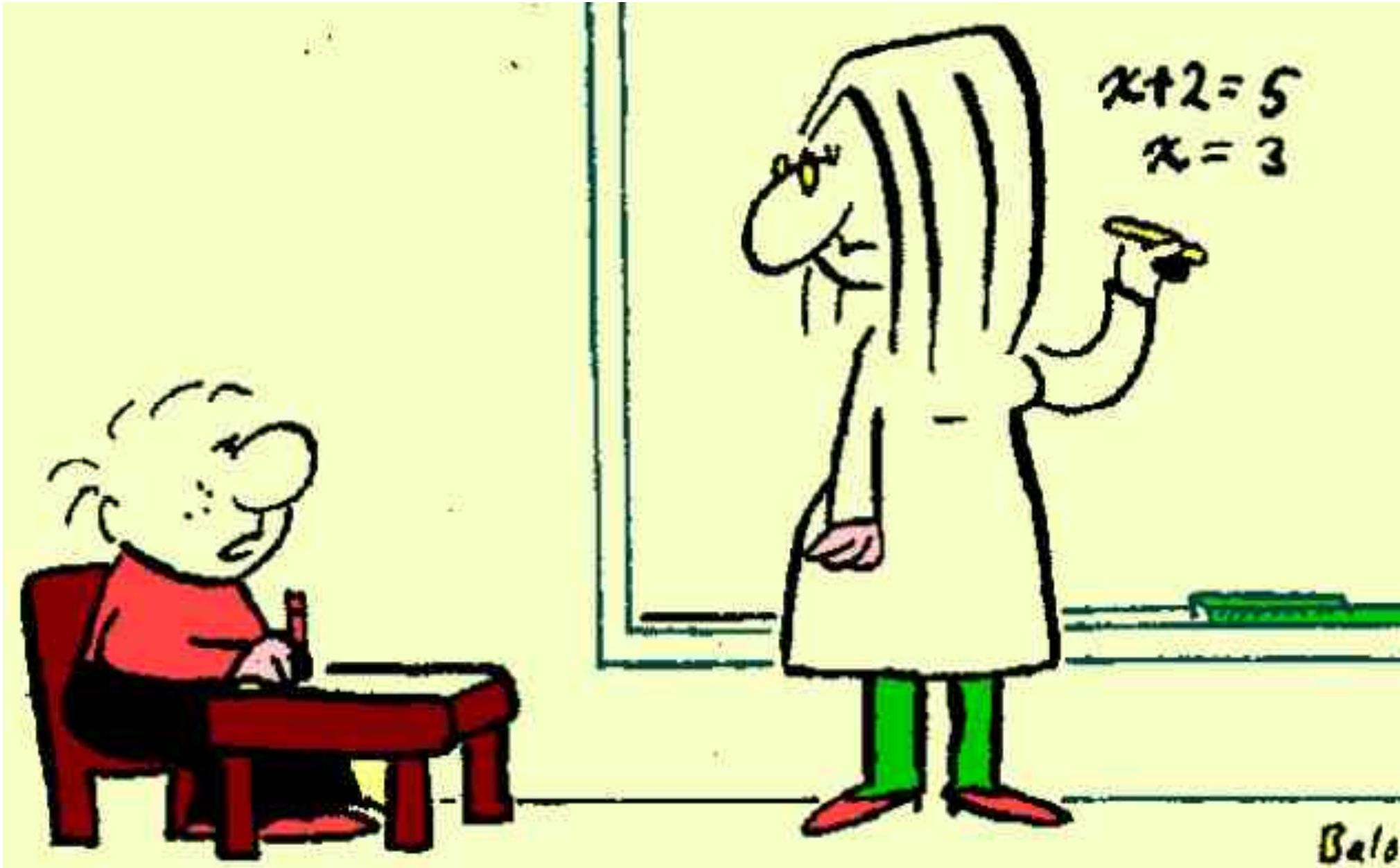
I've got my Students Sharing Their Mathematical Thinking— Now What?

Shari L. Stockero, Michigan Technological University
Laura R. Van Zoest, Western Michigan University
Keith R. Leatham, Brigham Young University

Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

- 4-year research project funded by the USA National Science Foundation (DRL-1220141, DRL-1220357, DRL-1220148)
- Co-Principal Investigators:
  - Keith R. Leatham, Brigham Young University
  - Blake E. Peterson, Brigham Young University
  - Shari L. Stockero, Michigan Technological University
  - Laura R. Van Zoest, Western Michigan University



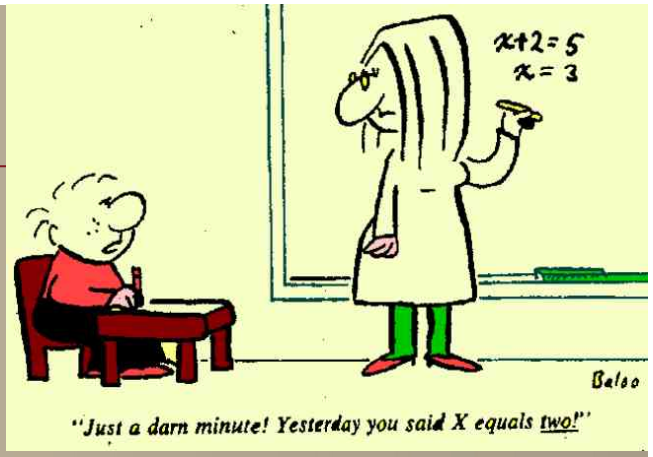
"Just a darn minute! Yesterday you said X equals <u>two!</u>"

Mathematical Opportunities in Student Thinking

Mathematically significant
pedagogical **O**pportunities
to build on **S**tudent
Thinking

Mathematical
Opportunities
in **S**tudent
Thinking

MOST Characteristics



SM: Yesterday x equaled 2 and today x equals 3.

Student Mathematical Thinking

Student Mathematics

Can the student mathematics be inferred?

Mathematical Point

Is there a mathematical point closely related to the student mathematics?

MP: A letter can be used to represent an unknown quality in an equation and can represent different values for different equations.

Mathematically Significant

Appropriate Mathematics

Is the mathematical point accessible to students with this level of mathematical experience, but not like to be already understood?

Central Mathematics

Is understanding the mathematical point a central goal for student learning in this classroom?

Pedagogical Opportunity

Opening

Does the expression of the student mathematics seem to create an intellectual need that, if met, will contribute to understanding the mathematical point of the instance?

Timing

Is now the right time to take advantage of the opening?

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education, 46*, 88-124.

MOSTs are opportunities…

Mathematical Opportunities in Student Thinking

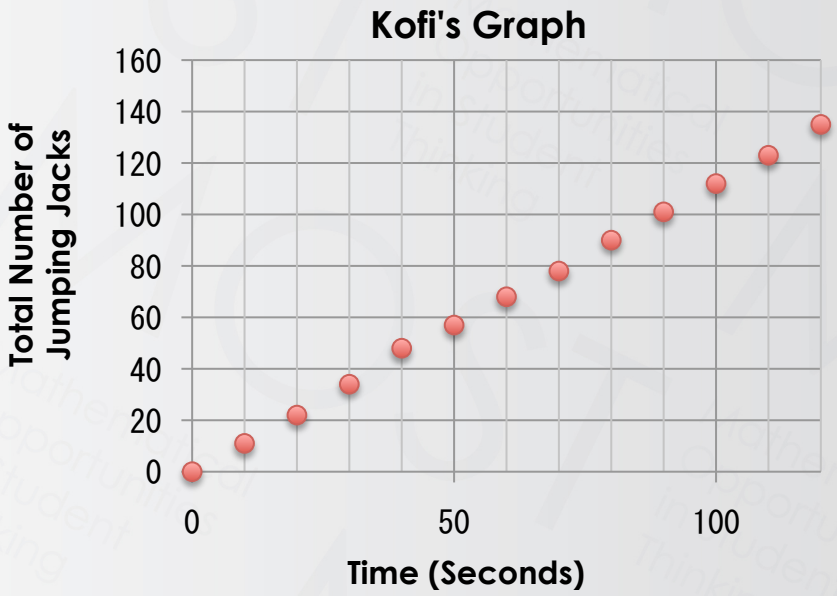
…for the teacher to make student mathematical thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

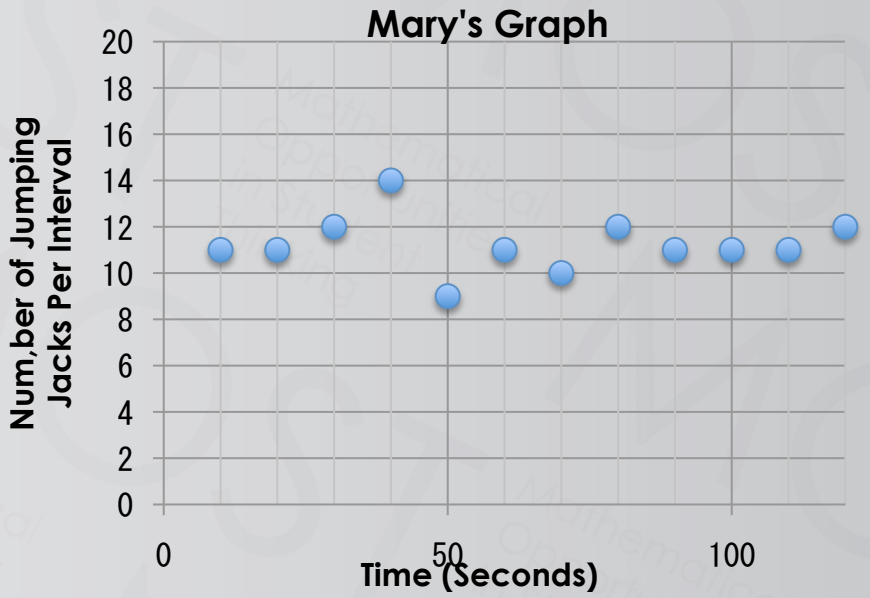
…to **build** on student thinking.

An Example

Mathematical Opportunities in Student Thinking

Jumping Jacks





Which student statement(s) in the transcript seem like MOSTs?

Transcript		Student Mathematics (SM)	Mathematical Point (MP)	Math Significant		Pedagogical Opportunity	
Teacher	Student			A	C	O	T
What is this graph [referring to "Mary's Graph"] for?							
Jumping jacks.		Mary's Graph relates to jumping jacks.	The labels on a graph tell you what is being represented in the graph.				
Did the jumping jack graphs we made look like this one?							
No.		The jumping jack graphs we made did not look like Mary's Graph.	None.				
What did the graphs look like?							
They were different.		The jumping jack graphs we made looked different from Mary's Graph.	None.				
What did they look like?							
They turned diagonally.		The jumping jack graphs we made were turned diagonally.	None.				
How is this different? Ours were, you know, how is this different? Number of jumping jacks?							
Isn't this one counted by intervals?		Isn't Mary's Graph different because it was counted by intervals?	Interpreting a graph requires that you understand the nature of the quantity the vertical axis represents.				
What does that mean? It says it on there—'per interval'—but what does that mean?							
Like um... I don't know but, right here, in 50 seconds she had only done 9 jumping jacks.		The graph shows that in 50 seconds she had only done 9 jumping jacks.	When measuring a quantity 'per interval' the dependent variable tells you how many units per interval (a rate) and not the total number of units.				

Transcript		Student Mathematics (SM)	Mathematical Point (MP)	Math Significant		Pedagogical Opportunity	
Teacher	Student			A	C	O	T
What is this graph [referring to "Mary's Graph"] for?							
Jumping jacks.		Mary's Graph relates to jumping jacks.	The labels on a graph tell you what is being represented in the graph. √				
Did the jumping jack graphs we made look like this one?							
No.		The jumping jack graphs we made did not look like Mary's Graph. √	None.				
What did the graphs look like?							
They were different.		The jumping jack graphs we made looked different from Mary's Graph. √	None.				
What did they look like?							
They turned diagonally.		The jumping jack graphs we made were turned diagonally. √	None.				
How is this different? Ours were, you know, how is this different? Number of jumping jacks?							
Isn't this one counted by intervals?		Isn't Mary's Graph different because it was counted by intervals?	Interpreting a graph requires that you understand the nature of the quantity the vertical axis represents.		√		
What does that mean? It says it on there—'per interval'—but what does that mean?							
Like um... I don't know but, right here, in 50 seconds she had only done 9 jumping jacks.		The graph shows that in 50 seconds she had only done 9 jumping jacks.	When measuring a quantity 'per interval' the dependent variable tells you how many units per interval (a rate) and not the total number of units.				√

MOSTs are opportunities…

Mathematical Opportunities in Student Thinking

…for the teacher to make student mathematical thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

…to **build** on student thinking.

Principles Underlying Productive Use of MOSTs

Mathematical Opportunities in Student Thinking

- The mathematics of the MOST is at the forefront.
- Students are positioned as legitimate mathematical thinkers.
- Students are engaged in sense making.
- Students are working collaboratively.

Is this Building?

Mathematical Opportunities in Student Thinking

Zac said, "Yesterday you said x equals two!"

Teacher Response 1: That's right. Because x represents an unknown, and the unknown is different in different equations, x can have different values.

Teacher Response 2: Zac is wondering why x was equal to 2 yesterday and now it's equal to 3 today.

Building …

Mathematical Opportunities in Student Thinking

- is a complex practice
- is not a single move
- must be a collection of moves

How would we recognize building if we saw it?

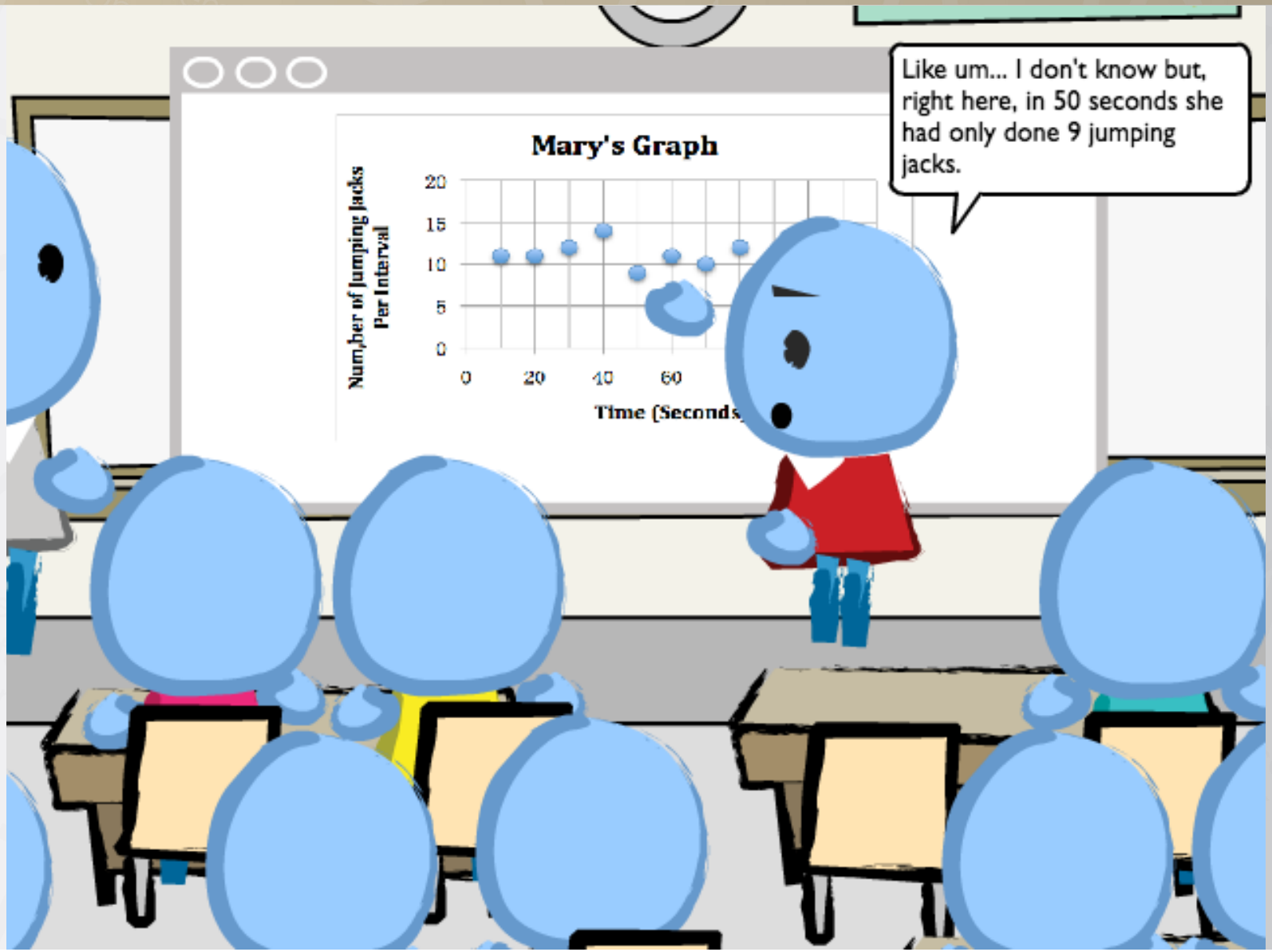
Building on Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

Example

Mathematical Opportunities in Student Thinking



Building on Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking

Building on Student Mathematical Thinking

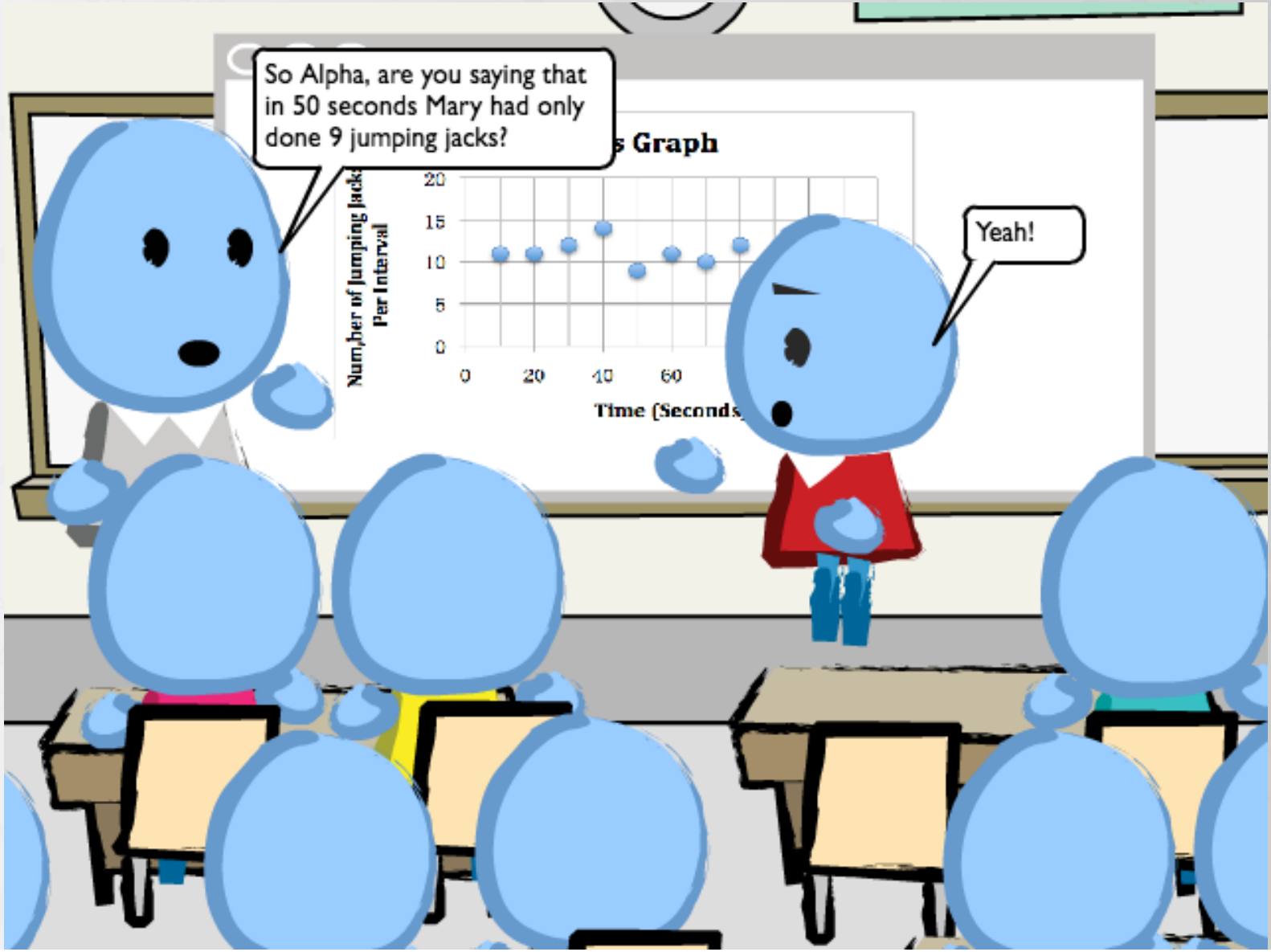
Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)

Example

Mathematical Opportunities in Student Thinking



Scenarios created with LessonSketch © The Regents of the University of Michigan (lessonsketch.org)

Building on Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear

Building on Student Mathematical Thinking

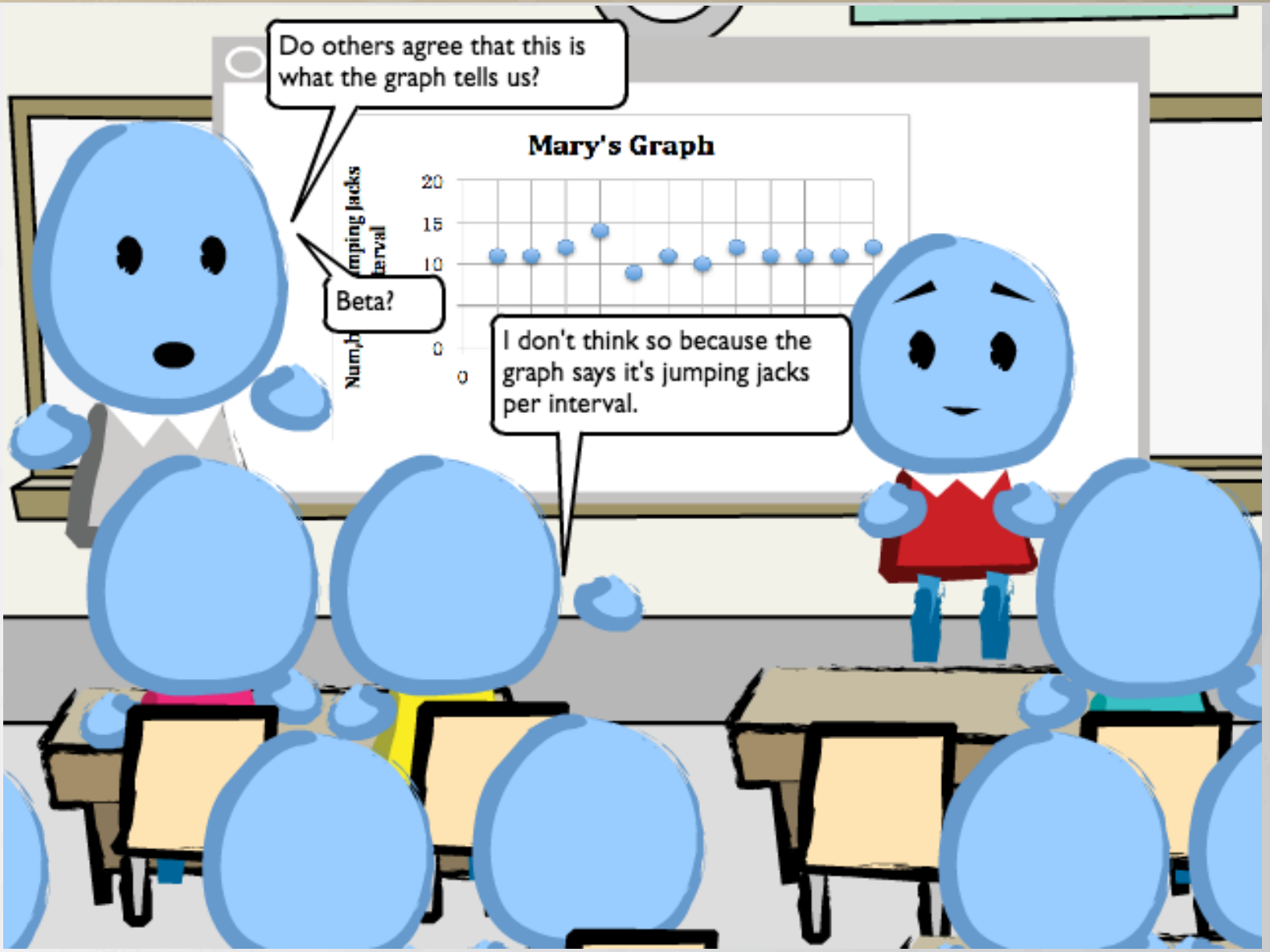
Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear (make precise)

Example

Mathematical Opportunities in Student Thinking



Scenarios created with LessonSketch © The Regents of the University of Michigan (lessonsketch.org)

Building on Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students

Building on Student Mathematical Thinking

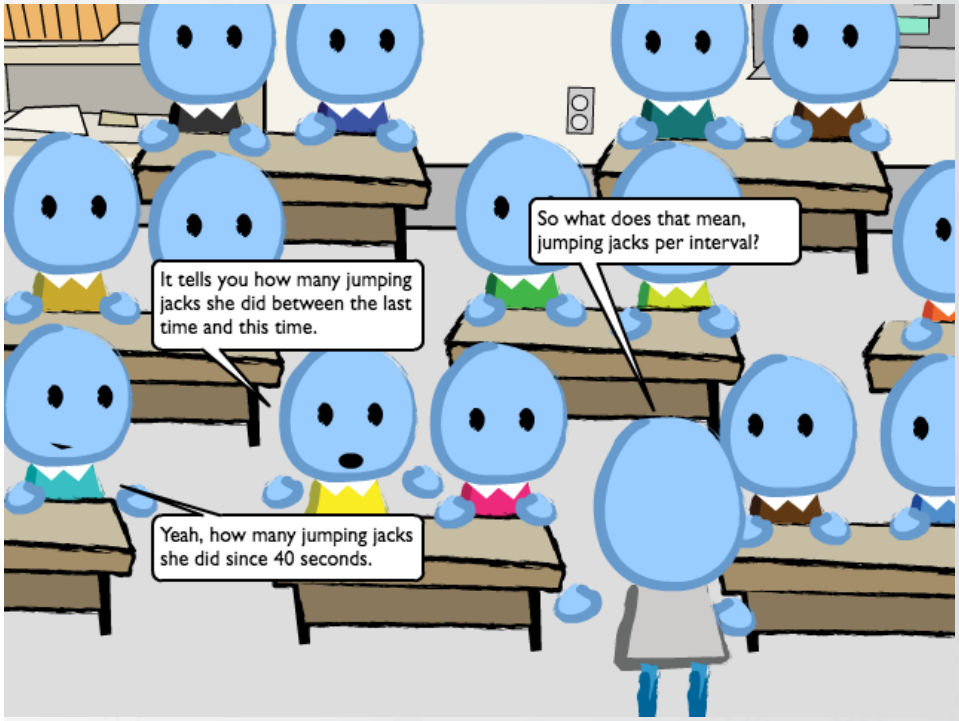
Mathematical Opportunities in Student Thinking

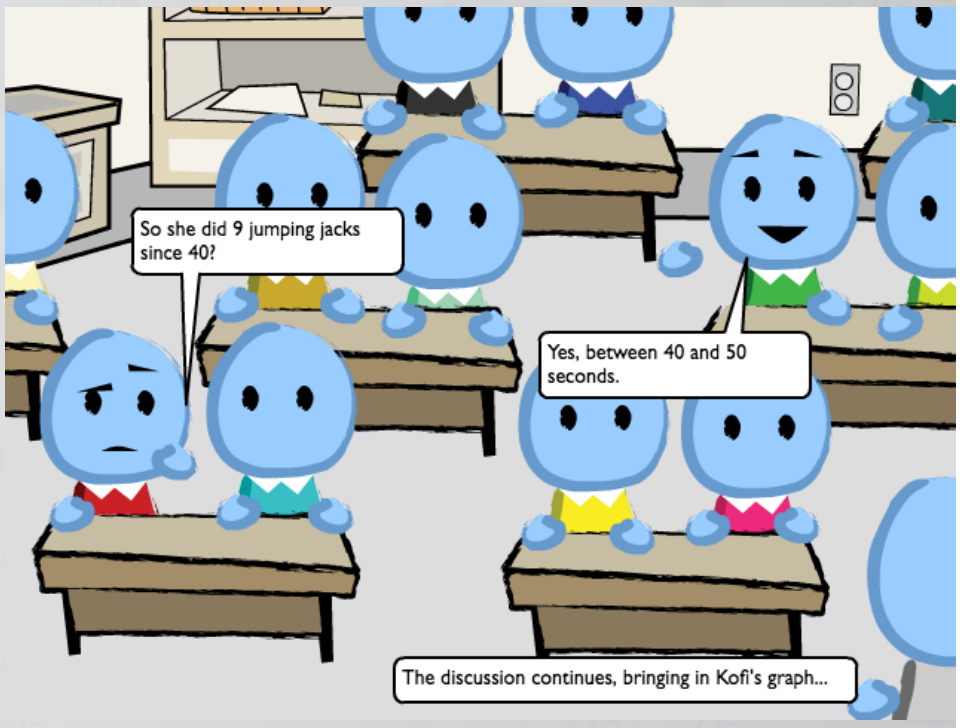
Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students (grapple toss)

Example

Mathematical Opportunities in Student Thinking





Building on Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students (grapple toss)
3. Orchestrate the students' process of making sense of the thinking

Building on Student Mathematical Thinking

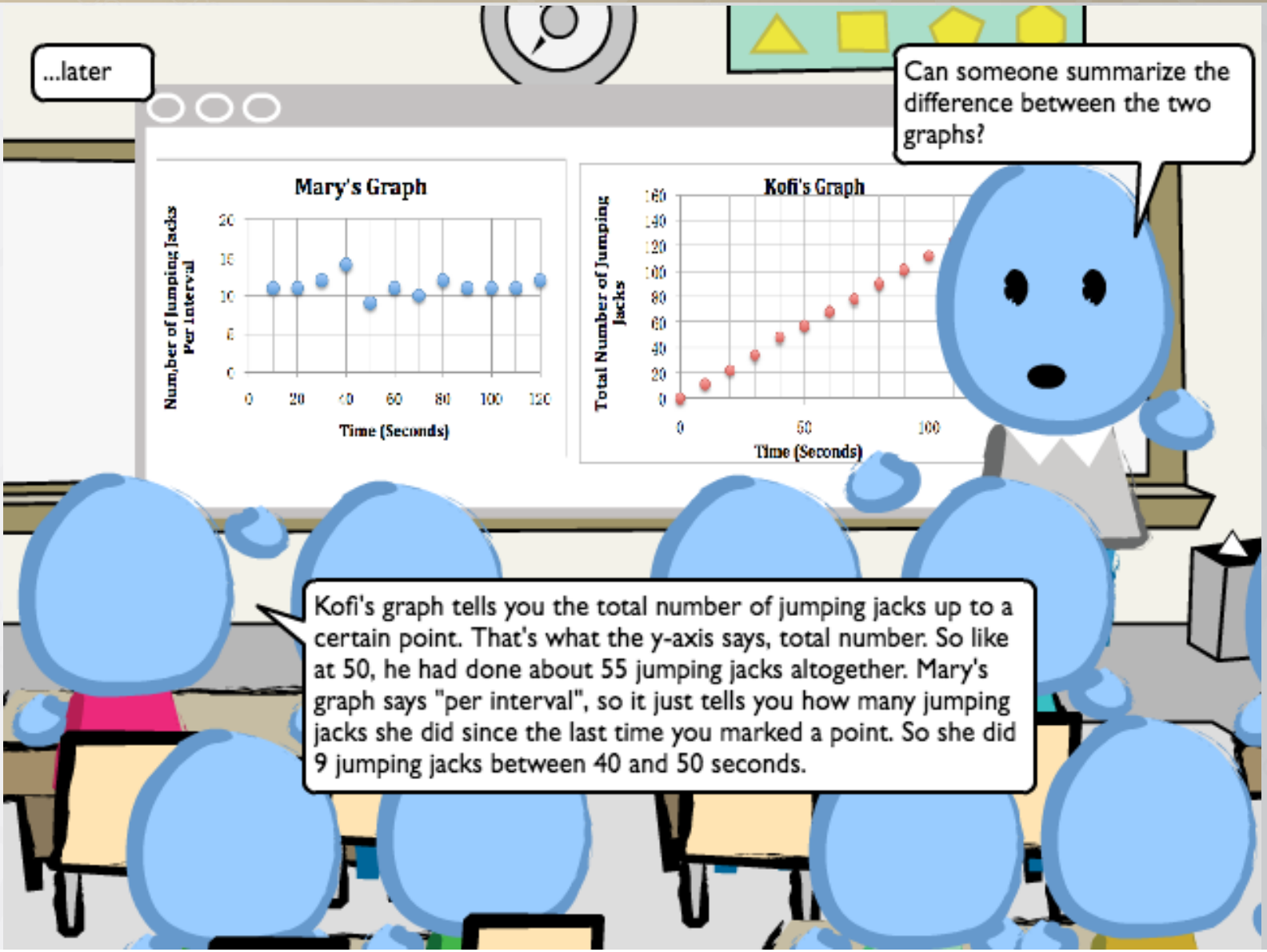
Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students (grapple toss)
3. Orchestrate the students' process of making sense of the thinking (orchestrate)

Example

Mathematical Opportunities in Student Thinking



Building on Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students (grapple toss)
3. Orchestrate the students' process of making sense of the thinking (orchestrate)
4. Facilitate the extraction of the important mathematical idea

Building on Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

Make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students (grapple toss)
3. Orchestrate the students' process of making sense of the thinking (orchestrate)
4. Facilitate the extraction of the important mathematical idea (make explicit)

Questions

Mathematical Opportunities in Student Thinking

- To what extent do these conceptualizations of MOST and building resonate with your experience?
- How do you see these conceptualizations as being useful in your practice as a mathematics teacher?

Contact Information

Mathematical Opportunities in Student Thinking

- LeveragingMOSTs.org

- Shari Stockero – stockero@mtu.edu
- Laura Van Zoest – laura.vanzoest@wmich.edu
- Keith Leatham – kleatham@mathed.byu.edu