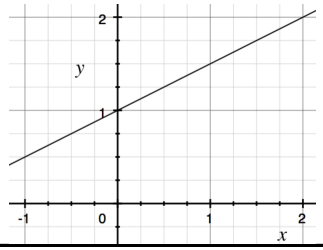
How We Can "Attend to Precision" in Classroom Mathematics Discussions

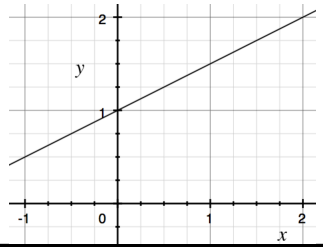
Keith R. Leatham
Blake E. Peterson
Lindsay M. Merrill
Brigham Young University

and…
Shari L. Stockero (M Tech)
Laura R. Van Zoest (WMU)
Sydney Bracha (BYU)
Allyson Derocher (BYU)

Supported by
Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking
a 4-year collaborative research project funded by the National Science Foundation (DRL-1220141, DRL-1220357, DRL-1220148)

Communication is Problematic

T=Teacher ***S=Student***		**Can you infer what the student means? Why or why not?**	**If not, how would you respond?**
The proportion $\frac{6}{4}=\frac{x}{10}$ is on the board. T: How could you solve this proportion for x?			
A	**S: By multiplying it by 10.**		
B	**S: By dividing.**		
The graph to the right is on the board. T: What is the slope of the line in this graph? 			
C	**S: It's 2.**		
Given $P=2.50V-500$ $V=600-500R$ the class was asked to (1) solve for P given $R=.25$ (2) solve for R given $P=625$ The class is now comparing the work involved in solving (1) and (2).			
D	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.**		
E	**S: In (1) you just do the equation instead of doing multiple step equations.**		

T=Teacher ***S=Student***		**Can you infer what the student means? Why or why not?**
The proportion $\frac{6}{4}=\frac{x}{10}$ is on the board. T: How could you solve this proportion for x?		
A	**S: By multiplying it by 10.**	
B	**S: By dividing.**	
The graph to the right is on the board. T: What is the slope of the line in this graph? 		
C	**S: It's 2.**	
Given $P=2.50V-500$ $V=600-500R$ the class was asked to (1) solve for P given $R=.25$ (2) solve for R given $P=625$ The class is now comparing the work involved in solving (1) and (2).		
D	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.**	
E	**S: In (1) you just do the equation instead of doing multiple step equations.**	

IMPRECISION

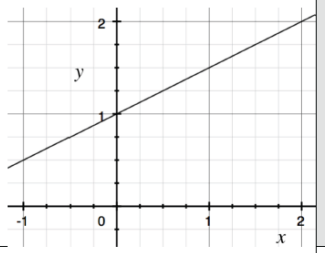
If a part of speech is used such that

- it can be interpreted in multiple viable ways,

- and the existence of those interpretations causes the overall meaning of the statement in which it occurs to be unclear,

then we refer to both the part of speech and the statement in which it occurs as *imprecise*.

Task: identifying imprecision

T=Teacher ***S=Student***		**Imprecision**
The proportion $\frac{6}{4}=\frac{x}{10}$ is on the board. T: How could you solve this proportion for x?		
A	**S: By multiplying it by 10.**	Subject- pronoun
B	**S: By dividing.** them. Divide *what* by *what*?	Object of verb- implied
The graph to the right is on the board. T: What is the slope of the line in this graph? 		
C	**S: It's 2.**	Not imprecise
Given $P=2.50V-500$ $V=600-500R$ the class was asked to (1) solve for P given $R=.25$ (2) solve for R given $P=625$ The class is now comparing the work involved in solving (1) and (2).		
D	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.** The opposite of *what*?	Object of adjective- implied
E	**S: In (1) you just do the equation instead of doing multiple step equations.**	Verb

Using pronouns is okay as long as it doesn't create imprecision!

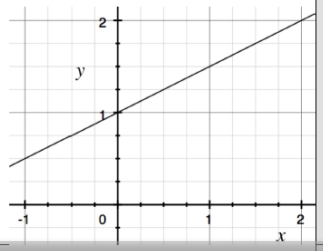
Imprecision

If a part of speech is used such that

- it can be interpreted in **multiple viable ways**,
- and the existence of those interpretations causes the **overall meaning** of the statement in which it occurs to be **unclear**,

then we refer to both the part of speech and the statement in which it occurs as *imprecise*.

Task: addressing imprecision

T=Teacher ***S=Student***		**How would you respond?**
The proportion $\frac{6}{4}=\frac{x}{10}$ is on the board. T: How could you solve this proportion for x?		
A	**S: By multiplying it by 10.**	When you say "multiply *it* by 10", what is the "it" that's being multiplied?
B	**S: By dividing.**	
The graph to the right is on the board. T: What is the slope of the line in this graph? 		
C	**S: It's 2.**	
Given $P=2.50V-500$ $V=600-500R$ the class was asked to (1) solve for P given $R=.25$ (2) solve for R given $P=625$ The class is now comparing the work involved in solving (1) and (2).		
D	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.**	So, for (2) you had to "do the opposite" of something. What was that *something*?
E	**S: In (1) you just do the equation instead of doing multiple step equations.**	

Based on our discussion, how would you respond to each instance?

Taking into account our definition of imprecision, how might we respond to D?

Addressing Imprecision

Honing in

Addressing Imprecision: Honing in

To address imprecision:

- Identify the imprecision
- Ask the student a clarification question that
  - Establishes what you *do* understand
  - *Is specific to the imprecise part of speech*

D	**S: For (1) you just had to plug in the number and for (2) you had to do the opposite.**	So, for (2) you had to "do the opposite" of something. What was that *something*?

Why do we address imprecision this way?

- Honors all the good stuff that the student said
- Highlights the part of speech that needs to be cleared up
  - Focuses your question

Why is it important to address imprecision?

Or put another way …

What are the ramifications of not addressing an instance of imprecision?

Consider the proportion problem $\frac{6}{4} = \frac{x}{10}$, when a student claims they can solve the proportion "by multiplying it by 10"

What are the rest of the students thinking about this student's claim *if the imprecision goes unaddressed*?

- They make an inference about the meaning of the claim.
  - *Their inference must be correct. (Regardless of the actual accuracy)*
  - Ramification: Might think they understand when they actually don't.
- They don't know what that student means.
  - *Everyone else must understand so I don't know what is going on.*
  - Ramification: Might think they don't understand when they actually do.

Consider the problem with two equations $P = 2.5V - 500$

$V = 600 - 500R$

What if some student interpret "do the equation" as

$$V = 600 - 500 \times .25$$

and other students interpret "do to equation" as

$$R = \frac{V - 600}{-500}$$

If the imprecision goes unaddressed, what happens in the classroom communication moving forward?

- Each group of students assumes their interpretation is correct.
- Subsequent statements are seen in light of each students' own interpretation
- Ramification: Inconsistent parallel conversations

Immediate Ramifications of Unaddressed Imprecision

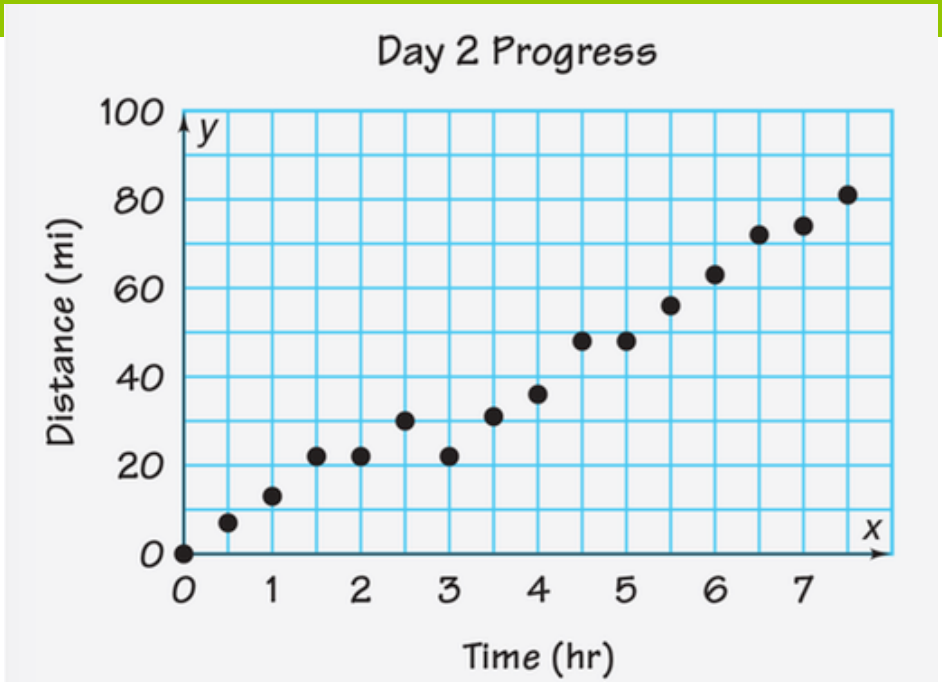
- Student Confusion
- Parallel Conversations

Long-term Implications

- Students disengage from class discourse
- Missed opportunities for the teacher to understand student mathematical thinking
- Student misconceptions develop or persist

Questions or Comments

Thank you!



T: "So this is stopped [pointing to the interval between 1.5 and 2]. Are we all okay with that? This is stopped."

S: "And then they went up."

T: "So what does that mean?"

S: "They went up."

T: "Does that mean they went up a hill? What do you mean by they went up?

S: [Student's making hand gestures indicating positive slope.]

As a teacher, be careful not to introduce imprecision

- Your own imprecise parts of speech
- Multiple questions