Productive Use of Student Mathematical Thinking is More than a Single Move

Blake E. Peterson, Brigham Young University
Laura R. Van Zoest, Western Michigan University
Shari L. Stockero, Michigan Technological University
Keith R. Leatham, Brigham Young University

Mathematical Opportunities in Student Thinking

Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

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- Co-Principal Investigators:
  - Keith R. Leatham, Brigham Young University
  - Blake E. Peterson, Brigham Young University
  - Shari L. Stockero, Michigan Technological University
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Mathematical Opportunities in Student Thinking

Mathematically significant
pedagogical **O**pportunities
to build on **S**tudent
Thinking

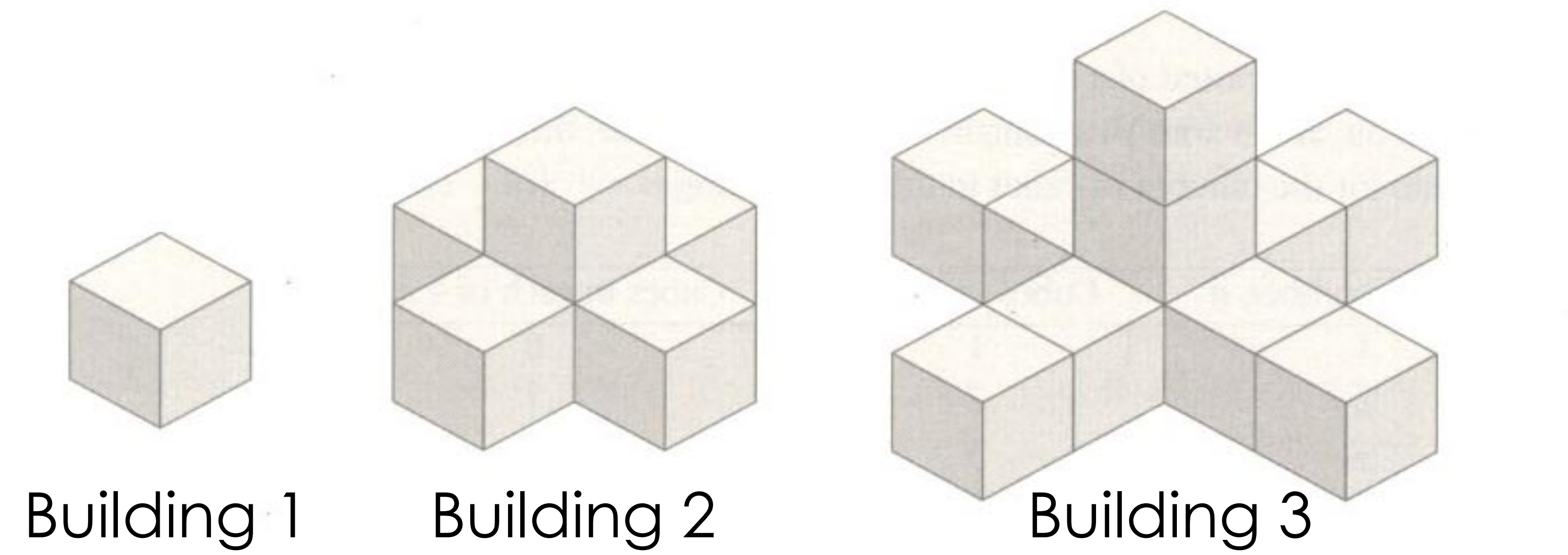
Mathematical **O**pportunities in **S**tudent **T**hinking

What is meant by "build on"?

- "*build on* student mathematical thinking" (Hill, Ball, & Schilling, 2008, p. 378)
- "*build on* student thinking during instruction" (Van Zoest & Stockero, 2012, p. 43)
- "*build on* and honor student mathematical thinking" (NCTM, 2014, p. 30)
- "*build on* student thinking and also advance important mathematical ideas" (Stein, Engle, Smith, & Hughes, 2008, p. 314)
- "whole class discussions that *build on* student thinking and guide the learning of the class" (NCTM, 2014, p. 35)

An Example

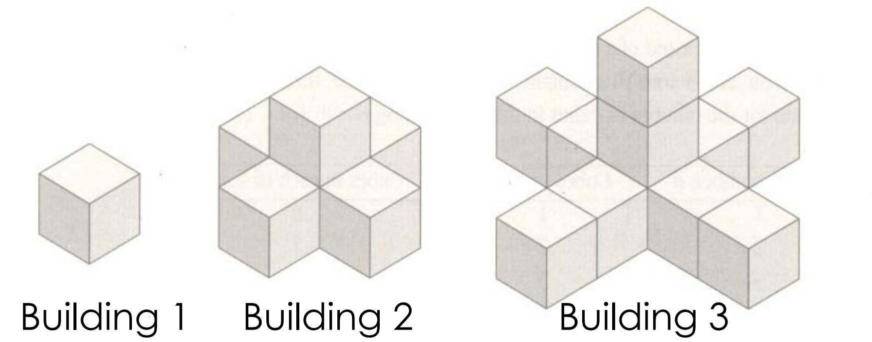
Mathematical Opportunities in Student Thinking



Study the sequence of cubes above. Assuming the sequence continues in the same way, how many cubes will there be in the *n*th building.

Adapted from "Counting Cubes", Lappan, Fey, Fitzgerald, Friel, & Phillips (2004). *Connected Mathematics - Say it with symbols: Algebraic Reasoning*

An Example



Student Solution A: The equation is $5n+1$ where *n* equals the length of 1 individual arm excluding the middle cube. You would multiply that by 5, because there are 5 arms, and then add 1 for the middle cube, and that'll give you the number of cubes.

Adapted from "The Case of Peter Dubno and the Counting Cubes Task", www.nctm.org/Conferences-and-Professional-Development/Principles-to -Action-Toolkit

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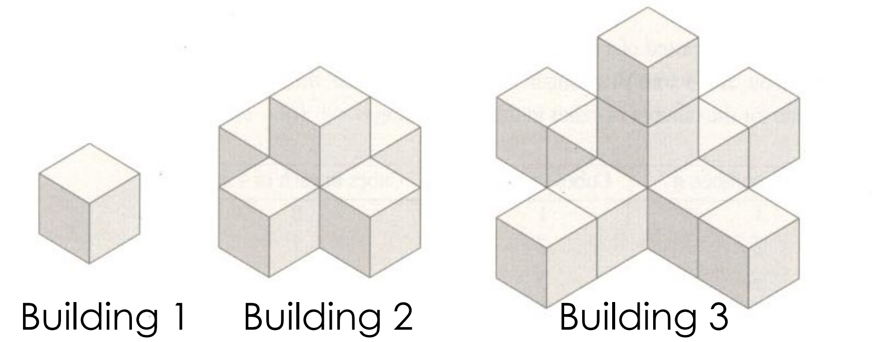

Student Solution B: We came up with $5n - 4$ There are 5 arms and *n* is the building number. Since each arm includes the middle block, we need to subtract 4 at the end.

Adapted from "The Case of Peter Dubno and the Counting Cubes Task", www.nctm.org/Conferences-and-Professional-Development/Principles-to -Action-Toolkit

Is this Building?

Mathematical Opportunities in Student Thinking

A teacher invites these two students to share their different solutions.

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Teacher Follow-up: "The expressions are different because the variable is defined differently in each case."

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A teacher invites these two students to share their different solutions.

Teacher Follow-up: "Both students have been able to count the number of blocks correctly for the 4^{th} building and the 17^{th} building but they have non-equivalent expressions. How is this possible?"

Building …

- is a complex practice
- is not a single move
- must be a collection of moves

How would we recognize building if we saw it?

Principles Underlying Productive Use of MOSTs

Mathematical Opportunities in Student Thinking

- The mathematics of the MOST is at the forefront.
- Students are positioned as legitimate mathematical thinkers.
- Students are engaged in sense making.
- Students are working collaboratively.

Our Definition of "Build"

Mathematical Opportunities in Student Thinking

Building on student mathematical thinking occurs when teachers make student mathematical thinking the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

Analyzing Teacher Practice

Mathematical Opportunities in Student Thinking

With this definition of building in mind identify which scenarios seem to be productive (or less productive) ways of building on student thinking. Justify your answer.

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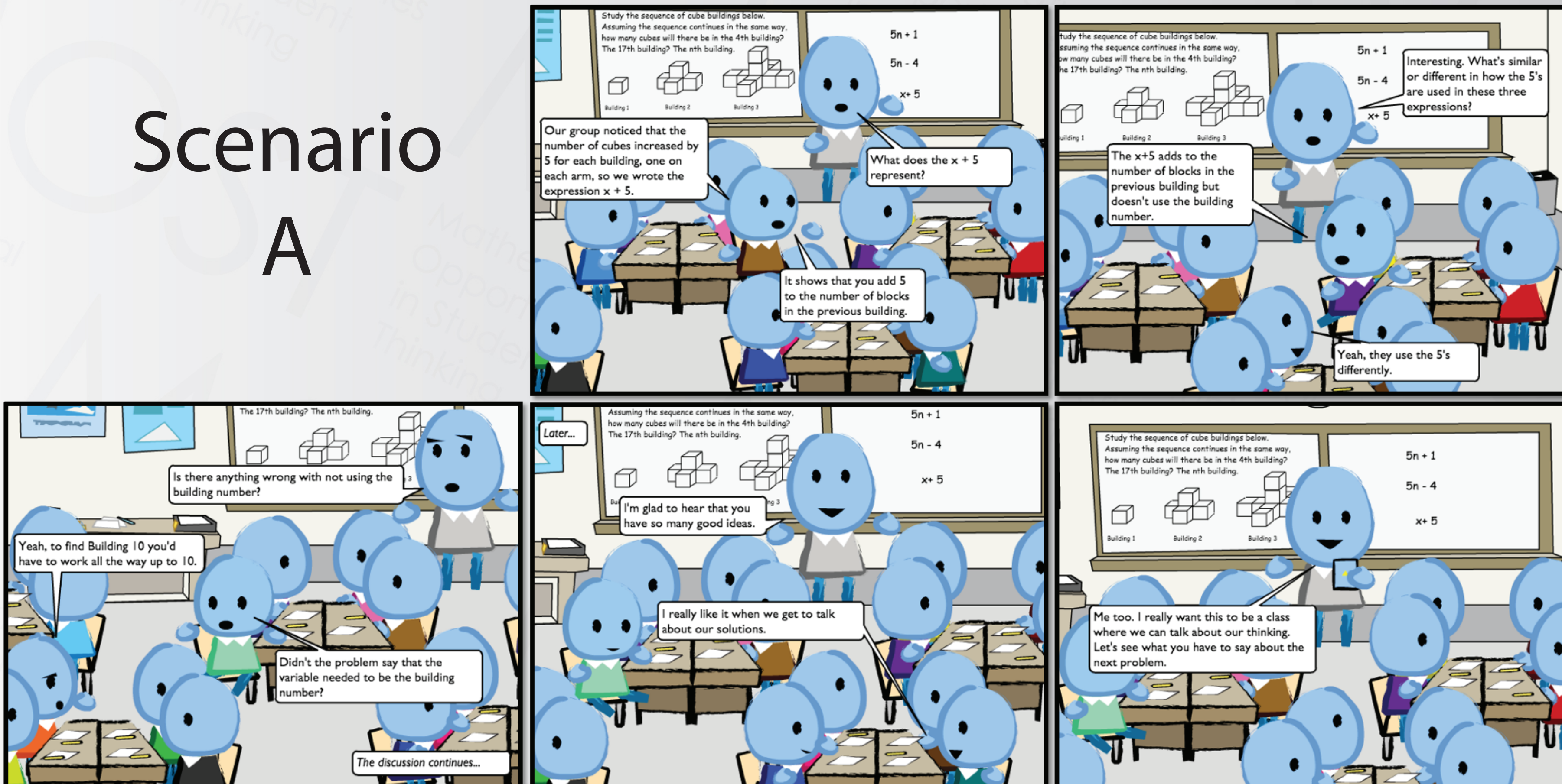
Analyzing LessonSketch Episodes

Mathematical Opportunities in Student Thinking

Scenario A

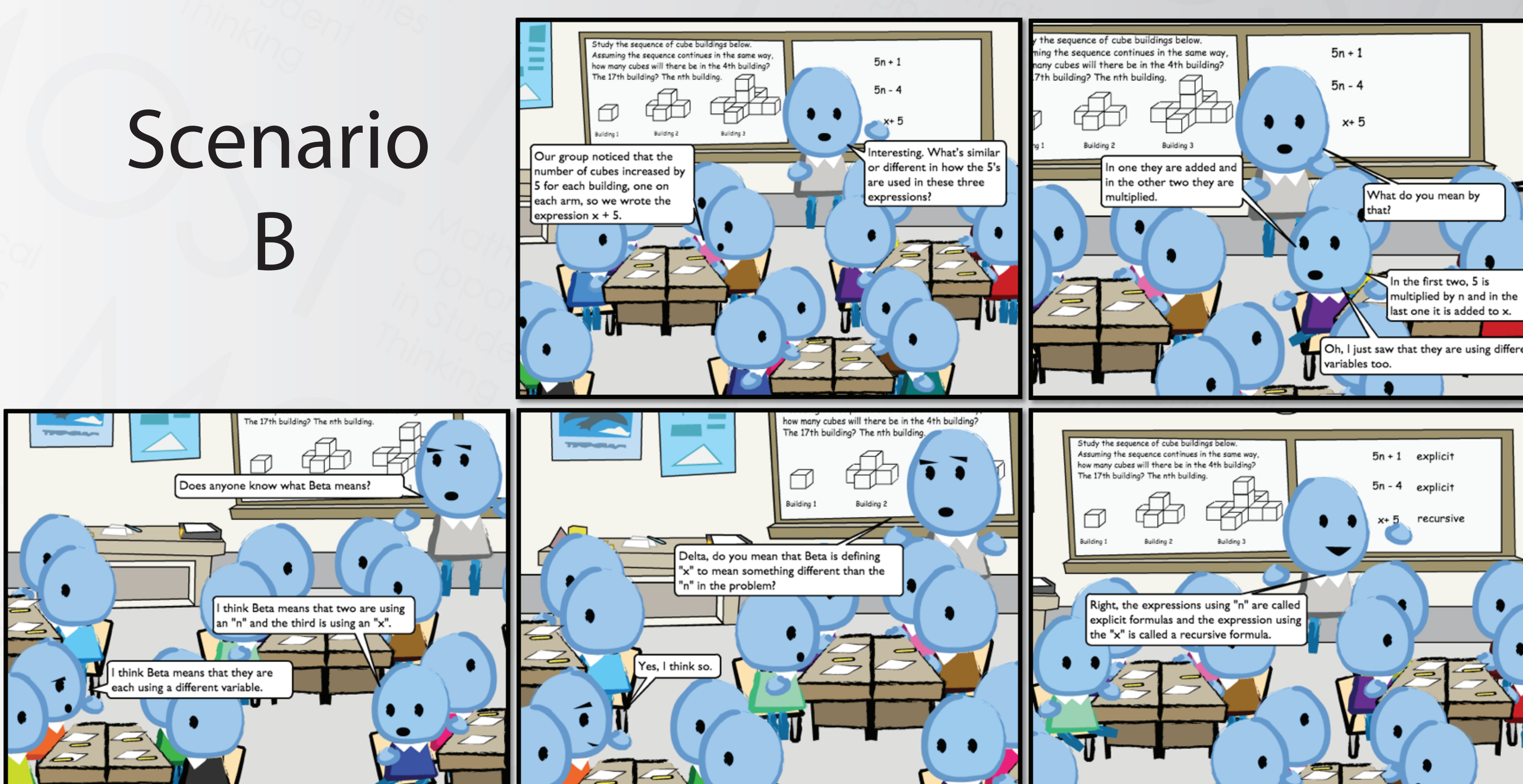


Scenarios constructed using LessonSketch (www.lessonsketch.org)

Analyzing LessonSketch Episodes

Mathematical Opportunities in Student Thinking

Scenario B



Scenarios constructed using LessonSketch (www.lessonsketch.org)

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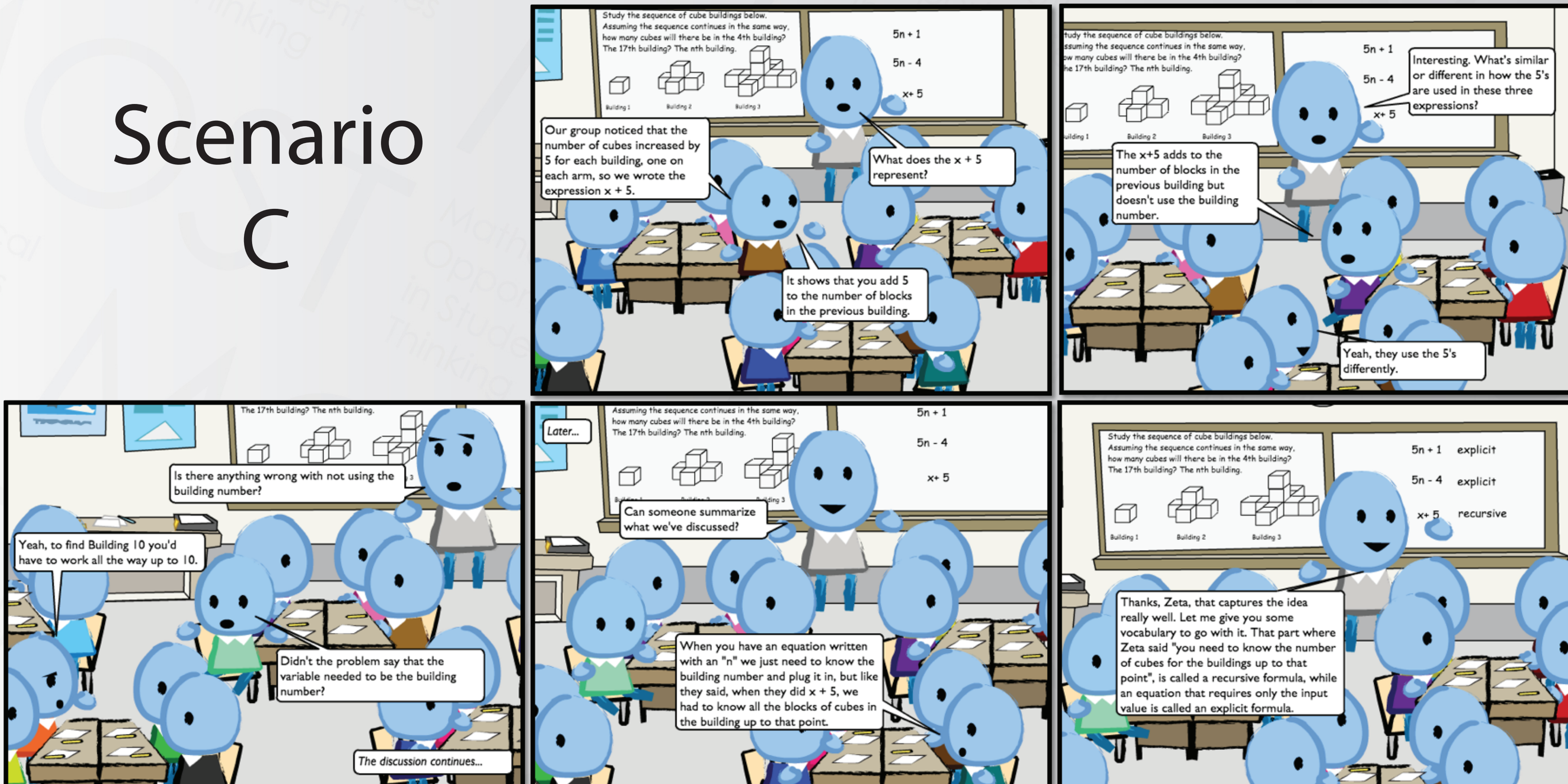
Mathematical Opportunities in Student Thinking

Scenario C

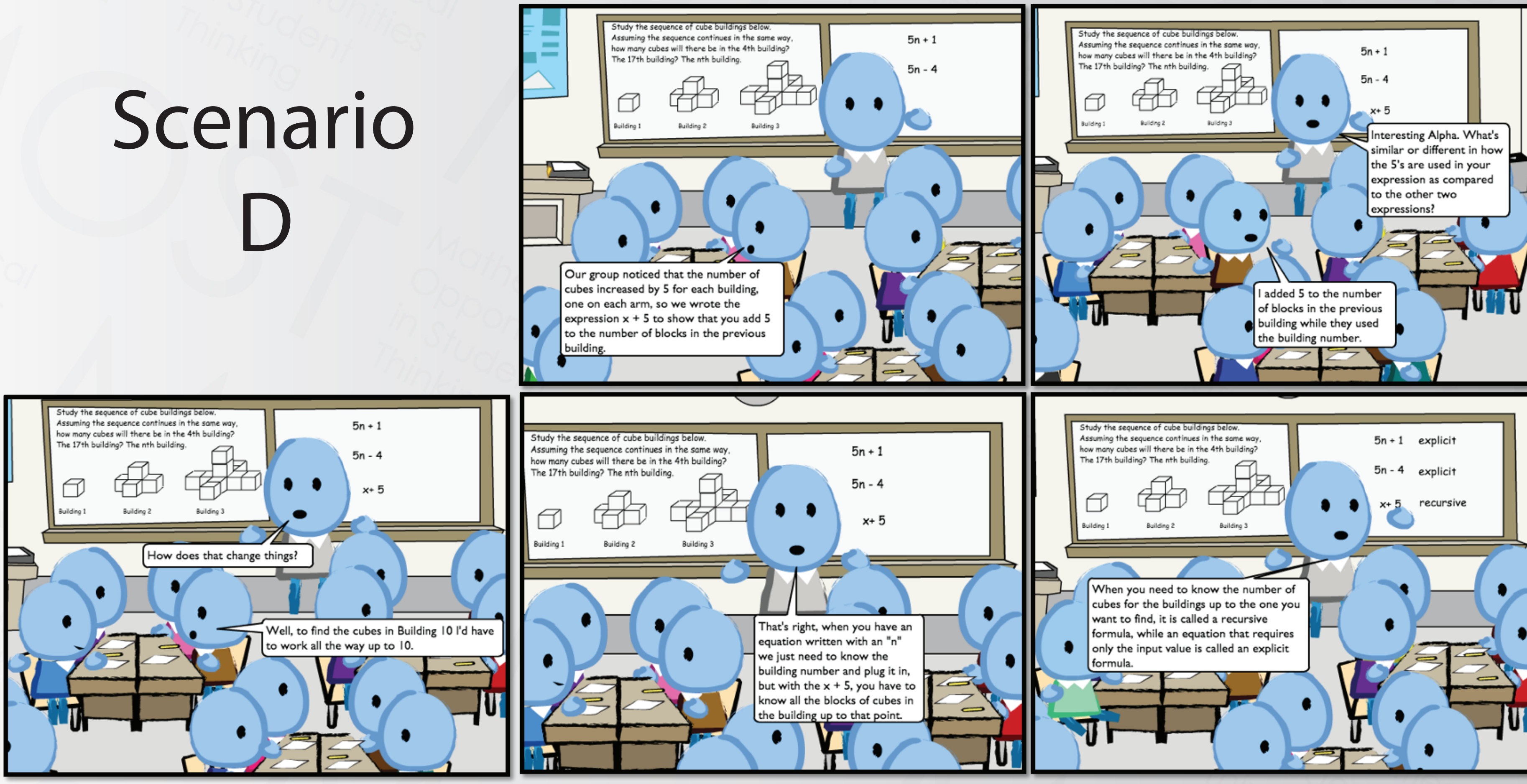


Scenarios constructed using LessonSketch (www.lessonsketch.org)

Analyzing LessonSketch Episodes

Mathematical Opportunities in Student Thinking

Scenario D



Scenarios constructed using LessonSketch (www.lessonsketch.org)

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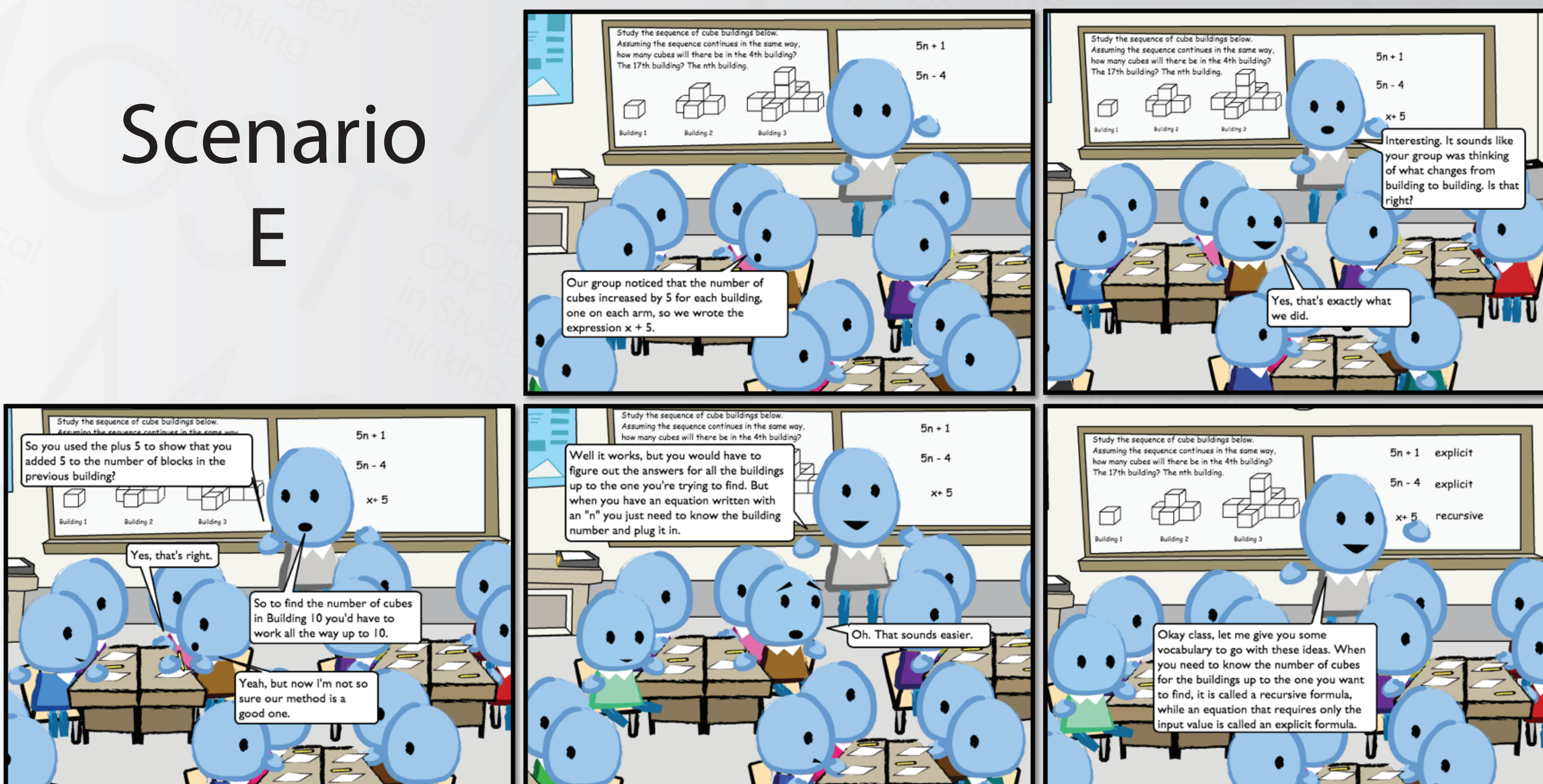
Mathematical Opportunities in Student Thinking

Scenario E

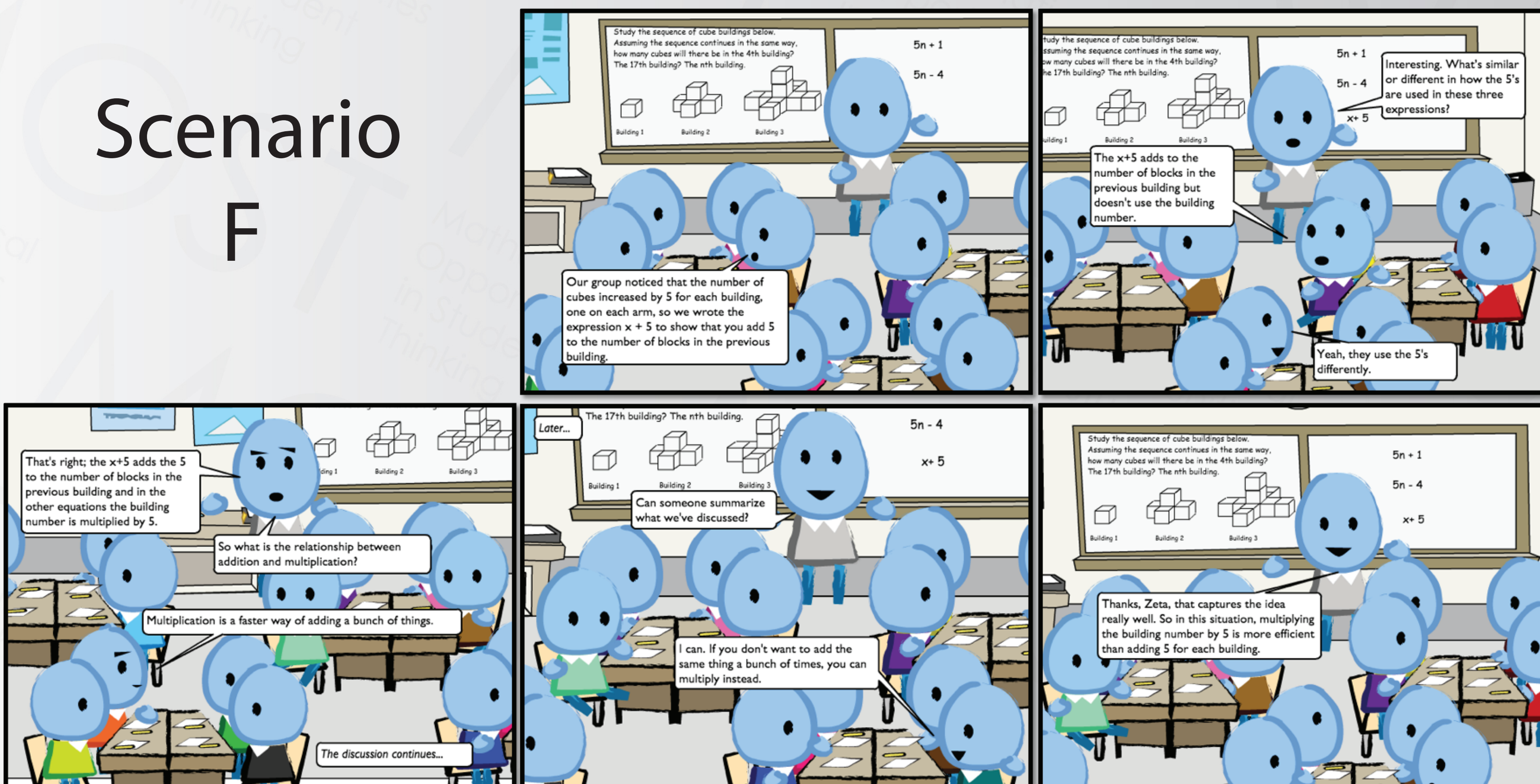


Scenarios constructed using Lesson*Sketch* (www.lessonsketch.org)

Analyzing LessonSketch Episodes

Mathematical Opportunities in Student Thinking

Scenario F



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Discussion Questions

- To what extent does this conceptualization of building resonate with your experience?
- How do you see this conceptualization as being useful in your practice as a mathematics teacher educator?
- What is the value of unpacking mathematics teaching practices to this level?

Contact Information

- LeveragingMOSTs.org

- Blake Peterson – blake@byu.edu
- Laura Van Zoest – laura.vanzoest@wmich.edu
- Shari Stockero – stockero@mtu.edu
- Keith Leatham – kleatham@mathed.byu.edu