What does it Mean to Build on Student Mathematical Thinking?

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Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

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Incorporating Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

- The mathematics education community encourages instruction that meaningfully incorporates students' mathematical thinking (e.g., NCTM, 2000, 2007)
- The benefits of such incorporation have been documented. (e.g., Fennema, et al., 1996; Stein & Lane, 1996)

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What does the literature say about incorporating student mathematical thinking?

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- *using* student mathematical thinking (e.g., Franke & Kazemi, 2001; Peterson & Leatham, 2009)
- *building on* student mathematical thinking (e.g., Hill, Ball, & Schilling, 2008; Van Zoest & Stockero, 2012)
- *attending to* the mathematical thinking (e.g, Feiman-Nemser & Remillard, 1996; Lampert et al., 2013)
- being "*responsive to* students and… their understanding" (Remillard, 1999, p. 331)
- *build on* students' prior or existing knowledge (Breyfogle & Herbel-Eisenmann, 2005; Carpenter et al., 1989).
- *build toward* an important mathematical idea (Stein, Engle, Smith, and Hughes, 2008)

Ways Teachers Incorporate Student Mathematical Thinking

Mathematical Opportunities in Student Thinking

- Attend to
- Pursue
- Use
- Build on
- Be responsive to

Ways Teachers Incorporate Student Mathematical Thinking

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- Attend to
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- Be responsive to

- Assess whether it is ok to move on
- Elicit student ideas
- Validate student ideas
- Have other students consider the thinking
- Engage in a discussion of the thinking

Our Use of "Use"

- In the context of classroom discourse
- Teacher doing something purposefully with student thinking
- Variations in the extent to which the teacher has to attend to what the student has said
  - Elicit
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Discussion

Mathematical Opportunities in Student Thinking

(a) Discuss the range of specific ways that a teacher might incorporate student thinking into instruction.

(b) For each of these ways,

- What does it accomplish?
- What is its value relative to its purpose?
- What is its value relative to the other ways?

Discussion

Elicit (e.g. merely have students share)
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Uses of Student Thinking

Mathematical Opportunities in Student Thinking

Use
Fill in the blank
IRE
Provoke/perturb student thinking
Follow incorrect student thinking
Have the class analyze student thinking
Compare two ideas

Uses of Student Thinking

Mathematical Opportunities in Student Thinking

Use	Purpose
Fill in the blank	
IRE	
Provoke/perturb student thinking	Shift mathematical authority? Get to know your students
Follow incorrect student thinking	Mathematical practices
Have the class analyze student thinking	Shift mathematical authority
Compare two ideas	Shift mathematical authority Understanding mathematics Valuing all ideas Knowing there are multiple ways Mathematical practices
Exit slips	Formative assessment
Eliciting	Get to know your students
	Honor student thinking

Uses of Student Thinking

Mathematical Opportunities in Student Thinking

Use	Purpose	Value (high, medium, low)
Fill in the blank	Cover curriculum, state tests, involve students	low
IRE	Cover curriculum, state tests, involve students	low
Provoke/perturb student thinking	Shift mathematical authority? Get to know your students	high
Follow incorrect student thinking	Mathematical practices	high
Have the class analyze student thinking	Shift mathematical authority	high
Compare two ideas	Shift mathematical authority Understanding mathematics Valuing all ideas Knowing there are multiple ways Mathematical practices	
Exit slips	Formative assessment	
Eliciting	Get to know your students	
	Honor student thinking	

Conclusions

- Productiveness—depends on purpose and what one values
- It is both possible and valuable to talk about these ideas and come up with a shared vocabulary

We chose "Build on"

Mathematical Opportunities in Student Thinking

Building on student mathematical thinking occurs when teachers make student mathematical thinking the object of consideration in order to engage students in making sense of that thinking to better understand an important mathematical idea.

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Contact Information

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