LeveragingMOSTs.org

Barriers to Building on Student Mathematical Thinking

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Supported by
Leveraging MOSTs: Developing a Theory of Productive Use of Student Mathematical Thinking
a 4-year collaborative research project funded by the US National Science Foundation (DRL-1220141, DRL-1220357, DRL-1220148)

Principles underlying productive use of student thinking

Mathematical Opportunities in Student Thinking

- Student mathematics is at the forefront
- Students are positioned as legitimate mathematical thinkers
- Students are engaged in sense making
- Students are working collaboratively

~aligned with NCTM, 2014, *Principles to Actions*

We see the *teaching practice of* ***building*** as simultaneously enacting these principles in response to in-the-moment student thinking.

Definition of building

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Building

To make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

Basic building blocks

Mathematical Opportunities in Student Thinking

- an **instance of student thinking**: an observable student action or small collection of connected actions (such as a verbal expression combined with a gesture)
- **student mathematics (SM)**: the articulation of a reasoned inference about what the student is saying mathematically in the instance
- **mathematical point (MP)**: the articulation of the most closely related mathematical idea that can be gained from considering the instance of student thinking

The MOST framework (Leatham et al., 2015) identifies instances of student thinking worth building on

Mathematical Opportunities in Student Thinking

The Teaching Practice of Building

To make student thinking an object of consideration for the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea.

0. Invite/allow students to share their mathematical thinking (elicit)
0.5 Recognize a MOST (recognize a MOST)
1. Make the object of consideration clear (make precise)
2. Turn the object of consideration over to the students with parameters that put them in a sense-making situation (grapple toss)
3. Orchestrate a whole-class discussion in which students collaboratively make sense of the object of consideration (orchestrate)
4. Facilitate the extraction and articulation of the mathematical point of the object of consideration (make explicit)

Teacher development experiments

Mathematical Opportunities in Student Thinking

- Worked individually with five teachers
  - Asked teachers to read about MOSTs
  - Supported teachers in planning
  - Observed lessons
  - Reflected together
- Focused on
  - helping teachers get better at recognizing and building on MOSTs
  - improving our own understanding of what this might entail

Barrier: Telling too much

Mathematical Opportunities in Student Thinking

- When students share their thinking they usually do so from their seats and Ken is the scribe at the board. He does this so he can:
  - make corrections
  - highlight certain points
    - **Ken**: I just want, I want the students to see the interesting thoughts that the other students are having so I like to point that out. So I don't know what kind of role that [is], "shower" I guess? The role of pointing out interesting ways that the students are thinking.
    - **Blake**: Tour guide?
    - **Ken**: Yeah exactly! Like a tour guide. But maybe pushing against some thinking also. Seeing if their thinking is correct or not. Or being able to have them justify their thinking. Yeah.
  - add further explanation

- Once the student thinking is shared, Ken rarely turns that thinking back to the class for consideration.
- He seems to be able to identify many MOSTs but feels his job is to be the "tour guide" and highlight the value of the student thinking that has been shared.

Barrier: Telling too much

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Telling too much – Awareness

Mathematical Opportunities in Student Thinking

Keith: Did you find yourself doing the same kind of pressing in the previous classes that you were doing when they were writing it all up there?

Ken: No. it was a little different. … When Kate was writing up her answer on the board, Sibilla was asking me, “Hey, why did she do this, this and this?” And that never happened when I was scribing, because I would make the decisions and then they’d wait till after to ask me or something else.

Telling too much – Awareness

Ken acknowledged that when he presses for clarification he does too much of the work.

- “I didn’t let her make the connection [for] the class. I made the connection for her to the class.”

Barrier: Student thinking on a tether

Mathematical Opportunities in Student Thinking

- As part of a warm-up, students were graphing $y = 3x - 4$.
- The teacher, Nan, asked Timothy to put his graph—that she thought was correct—on the board.
- Instead, the graph he drew represented $y = -4x + 3$.

Barrier: Student thinking on a tether
$y = 3x - 4$ (not $y = -4x + 3$)

Mathematical Opportunities in Student Thinking

Nan: Okay, so what does y-intercept mean?

Timothy: Uh, where it crosses, like where the line crosses the y-axis [gestures with his arm].

Nan: Okay and you're showing me with your hand y-axis [gestures with her arm]. Okay so does your line cross the y-axis, which you just showed me looks like this, does your line cross it at negative 4?

Timothy: No, ooh.

Nan: Where does his line cross it Erin? Erin where does his line cross the y-axis?

Erin: At positive 3.

Nan: At positive 3. Okay so what did he do? What did he mess up? Lonny what did he mess up on?

Lonny: He um, switched around-

Nan: He switched around his slope and y-intercept. Okay so instead we want to come down here to negative 4 and then if my slope is 3 what do I want to do next from there, Mickey?

Barrier: Student thinking on a tether

Mathematical Opportunities in Student Thinking

The Teaching Practice of Building

To make student thinking an object of consideration f[illegible] class in order to engage the class in making [illegible] thinking to better understand an importa[illegible]d.

I use [student thinking] more to like teach off of, versus turning it back over to the kids.

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Student thinking on a tether – Addressing the barrier

Mathematical Opportunities in Student Thinking

- Nan and I analyzed what she did in response to MOSTs and explored ways she might have grapple tossed.
  - We worked on articulating the student mathematics that the class could grapple with.
  - In her classroom she worked on recording verbal comments on the board or projector.

Student thinking on a tether – Awareness

- In that process of writing [the student mathematics] down, I'm slowing myself down so that I truly understand what the students are saying and then it helps me to know how to best use what they've said next. Where if I don't write it down I'm still on that train of like, okay, I'll repeat it but I just keep going.

- [By analyzing the student thinking] you get better and better at knowing how to turn it around and say hey, okay, let's look at this method together as a class and let's check and see does this work in all situations and then turn it back to the students … [Having the students grapple with a MOST] just creates like a higher level of thinking in your room.

Barrier: Focusing on important mathematics

Mathematical Opportunities in Student Thinking

The task is to find the midpoint of two points. Students are considering this question using the points (1,1) and (13, 4). A right triangle has been drawn on the board whose vertices are the given points and the point (13, 1).

Eli comes to the board and explains: "Half of 12 is 6 so that's 1, 2, 3, 4, 5, 6 (counting along bottom side of triangle), [marks point at 7], and half of 3 is 1.5, so [marks point at 2.5 on vertical side of triangle]." He then draws horizontal and vertical lines through the two marked points, marks the intersection, and says, "So that's the midpoint".

Tia: Will that method work for any triangle I give you?

Several students: No.

Tia: Why not?

Alec: Cause they're not exact. [inaudible] like the grid is 1, 2, 3, 4…but some grids are like 20, 40, 60, 80.

Tia goes on to ask a series of questions about using different scales on the horizontal axis. She indicates that Eli's method works, but does not explore why.

Barrier: Focusing on important mathematics

Mathematical Opportunities in Student Thinking

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Focusing on important mathematics – Addressing the barrier

Mathematical Opportunities in Student Thinking

- Pushing Tia to anticipate how students might approach tasks so she was better prepared to identify potential MOSTs.
- Working with Tia to articulate the student mathematics and the mathematical point of MOSTs that surfaced during her instruction.

Focusing on important mathematics – Awareness

Mathematical Opportunities in Student Thinking

"I definitely still want to work on my content knowledge. A lot of times when I have a student moment happen, and I'm not familiar with the comment, it's like deer in the headlights, like, ah, how do I do this? What do I do? And a lot of times I revert right back to teacher-led instruction."

Barrier: Telling too little

Mathematical Opportunities in Student Thinking

- Three snapshots
  - Another what?
  - Common difference vs common ratio
  - Secret generalization
- **Keith**: “I think you may be avoiding telling when it would actually be important to do so.”

Barrier: Telling too little

Mathematical Opportunities in Student Thinking

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Telling too little – Addressing the barrier

Mathematical Opportunities in Student Thinking

- Discuss specific instances from practice
- Name barrier
- Relate to the building practice
- Read and discuss *Beyond Being Told Not to Tell* (Chazan & Ball, 1999)

Telling too little – Awareness

Mathematical Opportunities in Student Thinking

"I've learned… For me, it would be really easy to put a problem on the board and just say, "You do this, you just… you do that, you do this." So something that I've learned is that…. I saw…. doing exactly that as being like this plague or something, and I wanted to get as far away from it as possible. And in doing so I've left some important things out of my teaching. And now… So I don't know if this will answer the question, but I feel like something else that is really valuable that has come from it is that you don't have to... You can grab this thing that you're running away from. You can take this thing too. That 'what' I was running away from isn't all, like, bad. Like, its not. Yeah."

"This whole experience has better defined my role. Which has been so helpful, because I've questioned that so much. 'What am I doing? What do I do? So, okay, get they're thinking out there, but…' And I think I started seeing… Yeah, I'm not supposed to lead them to a certain place, but I'm not supposed to just let all these ideas come and then just say, 'Okay, so now what? So now what? So now what?' But, rather, organize these things. And it's something that I forgot, I guess. I mean, I've been taught that…. Sometimes it seems daunting to me, as far as orchestrating goes."

Barriers

Mathematical Opportunities in Student Thinking

- Telling too much (Ken)
- Tethering student thinking (Nan)
- Focusing on important mathematics (Tia)
- Telling too little (Ian)

Building Subpractices

0. Invite/allow students to share their mathematical thinking (elicit)
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Discussion questions

Mathematical Opportunities in Student Thinking

1. What are your reactions to the barriers these teachers encountered?
   - Telling too much (Ken)
   - Tethering student thinking (Nan)
   - Focusing on important mathematics (Tia)
   - Telling too little (Ian)
2. What other barriers might inhibit teachers' use of the building practice?
3. What are possible ways to overcome identified barriers?

Contact information

Mathematical Opportunities in Student Thinking

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